



# A Multi-Index Multivariate Drought Monitoring Framework

Amir AghaKouchak, Zengchao Hao and  
Navid Nakhjiri

*University of California, Irvine*

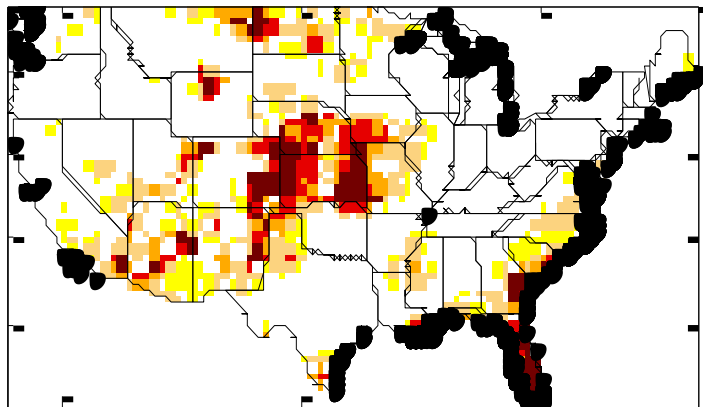


# Multi-Index Drought Monitoring

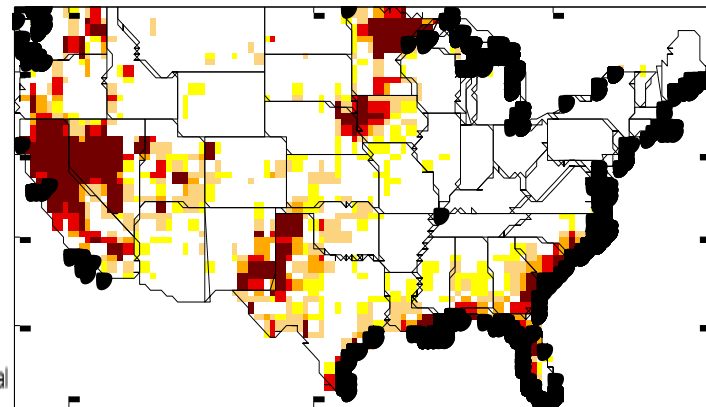
Different drought indices based on different climate variables (e.g., Precipitation, soil moisture):

- Standardized Precipitation Index (SPI)
- Standardized Soil Moisture Index (SSI)
- Standardized runoff Index (SRI)
- Palmer Drought Severity Index (PDSI)

## SPI 2012-1



## SSI 2012-1

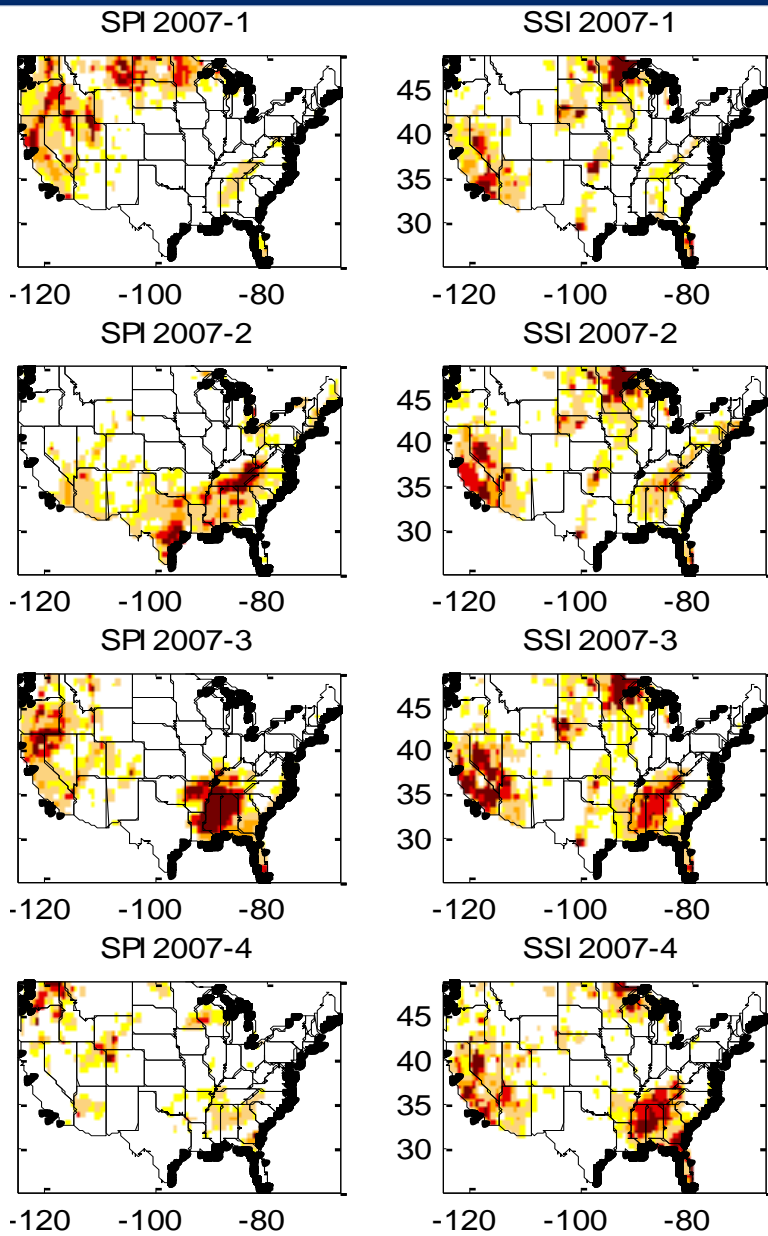


*Intensity:*  
D0 Abnormally Dry  
D1 Drought - Moderate  
D2 Drought - Severe  
D3 Drought - Extreme  
D4 Drought - Exceptional










# Multi-Index Drought Monitoring



Intensity:

-  D0 Abnormally Dry
-  D1 Drought - Moderate
-  D2 Drought - Severe
-  D3 Drought - Extreme
-  D4 Drought - Exceptional

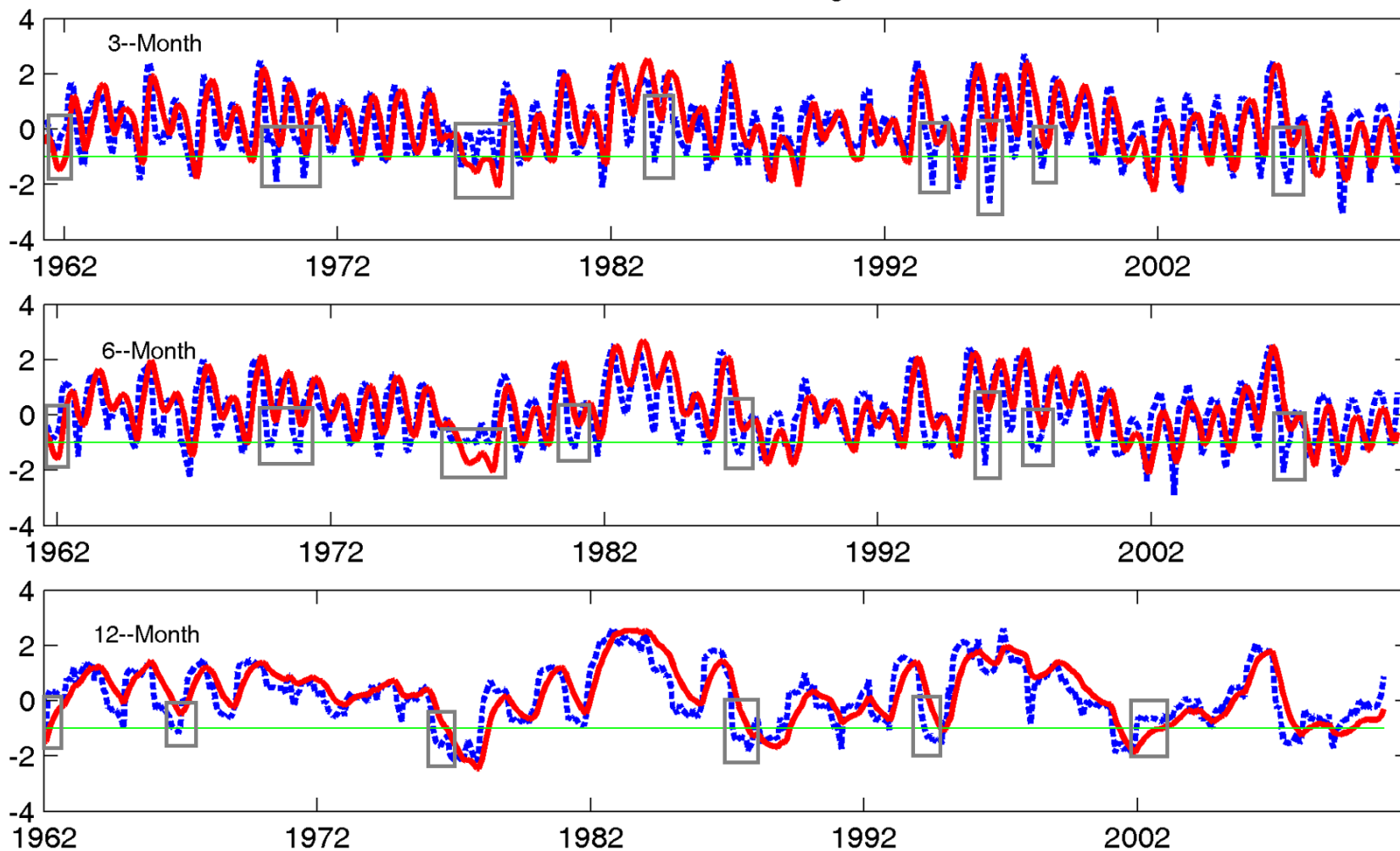




# Multi-Index Drought Monitoring

3-, 6- and 12-month drought conditions based on Standardized Precipitation Index (SPI) and Standardized Soil Moisture Index (SSI) for Climate Division 3 in California.

----- SPI    — SSI    — Drought Threshold -1





## A Multi-Index Approach Using a Joint Distribution Function

$$p = P(X \leq x, Y \leq y)$$

$$p = C[F(X), G(Y)]$$

C is the copula and F (X) and G (Y) are the marginal cumulative distribution functions of precipitation (X) and soil moisture (Y), respectively



# Multi-Index Drought Monitoring

## A Multi-Index Approach Using a Joint Distribution Function

$$p = P(X \leq x, Y \leq y)$$

$$p = C[F(X), G(Y)]$$

C is the copula and F (X) and G (Y) are the marginal cumulative distribution functions of precipitation (X) and soil moisture (Y), respectively

### Multivariate Standardized Drought Index (MSDI)

$$MSDI = \phi^{-1}(p)$$

where  $\phi$  is the standard normal distribution function.





# Multi-Index Drought Monitoring

## Deriving the Joint Distribution of Precipitation and Soil Moisture

$$p = C[F(X), G(Y)]$$

### Non-Parametric Models

Yue et al., 1999

$$P(X \leq x, Y \leq y) = \left( \sum_{i=1}^k \sum_{j=1}^k n_{ij} - 0.44 \right) (m + 0.12)^{-1}$$

where  $n$  is the length of the observations; and  $n_{ij}$  is the number of occurrences of the pair  $(x_i, y_i)$  for  $x_i \leq x_k$ ,  $y_i \leq y_k$  with  $1 \leq i \leq k$ .





# Multi-Index Drought Monitoring

## Deriving the Joint Distribution of Precipitation and Soil Moisture

$$p = C[F(X), G(Y)]$$

### Parametric Models

Nelson 2006; Joe 1997; AghaKouchak 2010

$$C_{v,\rho}(u_1, u_2) = t_{v,\rho}^2(t_{v,\rho}^{-1}(u_1), t_{v,\rho}^{-1}(u_2))$$

$$t_{v,\rho}^2(x) = \frac{1}{\sqrt{\det \rho}} \frac{\Gamma(\frac{v+n}{2})}{\Gamma(\frac{v}{2})(\pi v)^{n/2}} \int_{-\infty}^{x_1} \int_{-\infty}^{x_2} \frac{dx}{(1 + \frac{x' \rho^{-1} x}{v})^{(v+n)/2}}$$







## A Multi-Index Approach Using a Joint Distribution Function

$$p = P(X \leq x, Y \leq y)$$

$$p = C[F(X), G(Y)]$$

C is the copula and F (X) and G (Y) are the marginal cumulative distribution functions of precipitation (X) and soil moisture (Y), respectively

### Multivariate Standardized Drought Index (MSDI)

$$MSDI = \phi^{-1}(p)$$

where  $\phi$  is the standard normal distribution function.



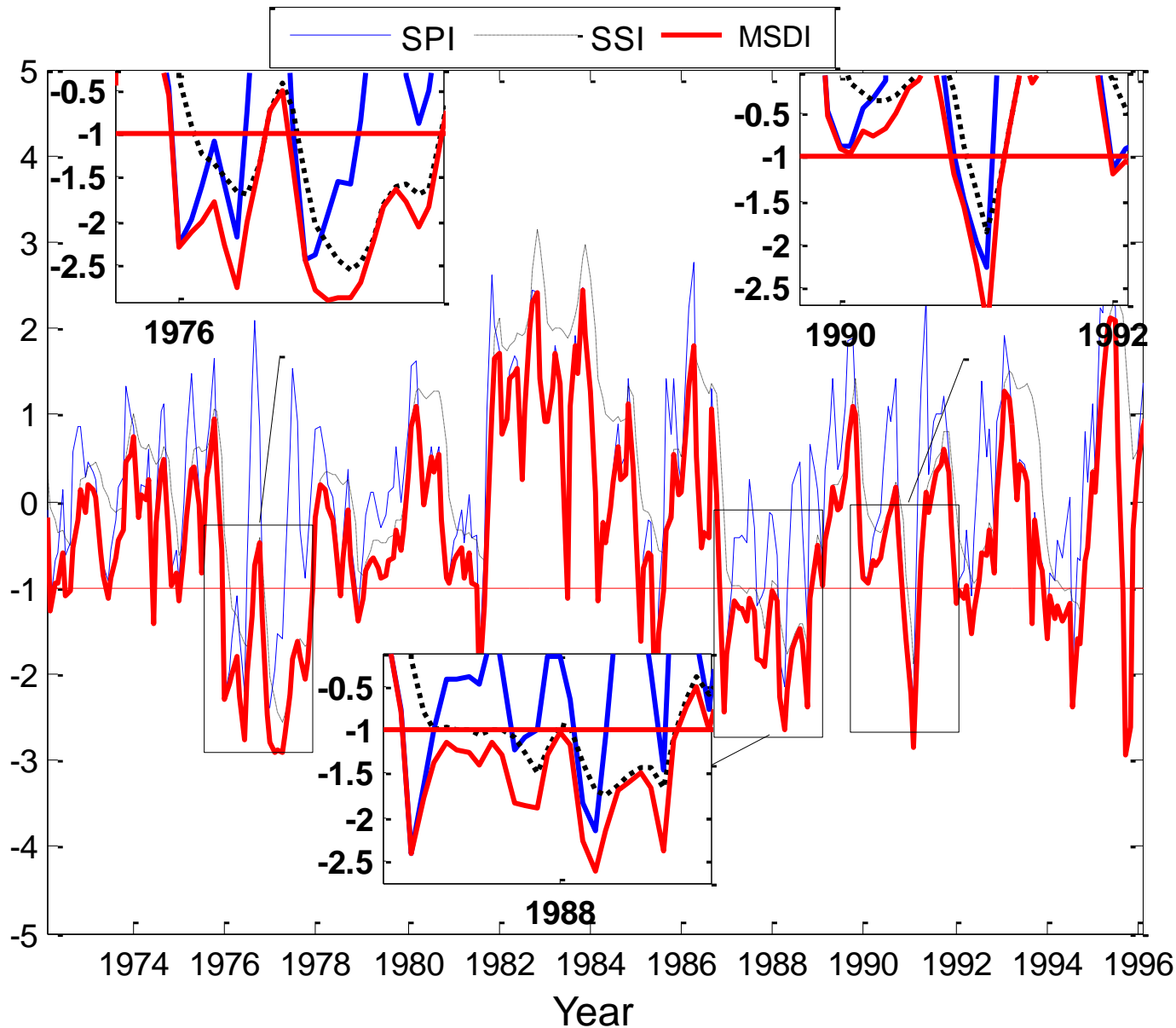


# Multi-Index Drought Monitoring

Monthly precipitation and soil moisture from 1/1932 to 12/2010

Precipitation : NCDC  
Soil Moisture: CPC

3-month SPI, SSI, MSDI for Climate Division 3 in California.



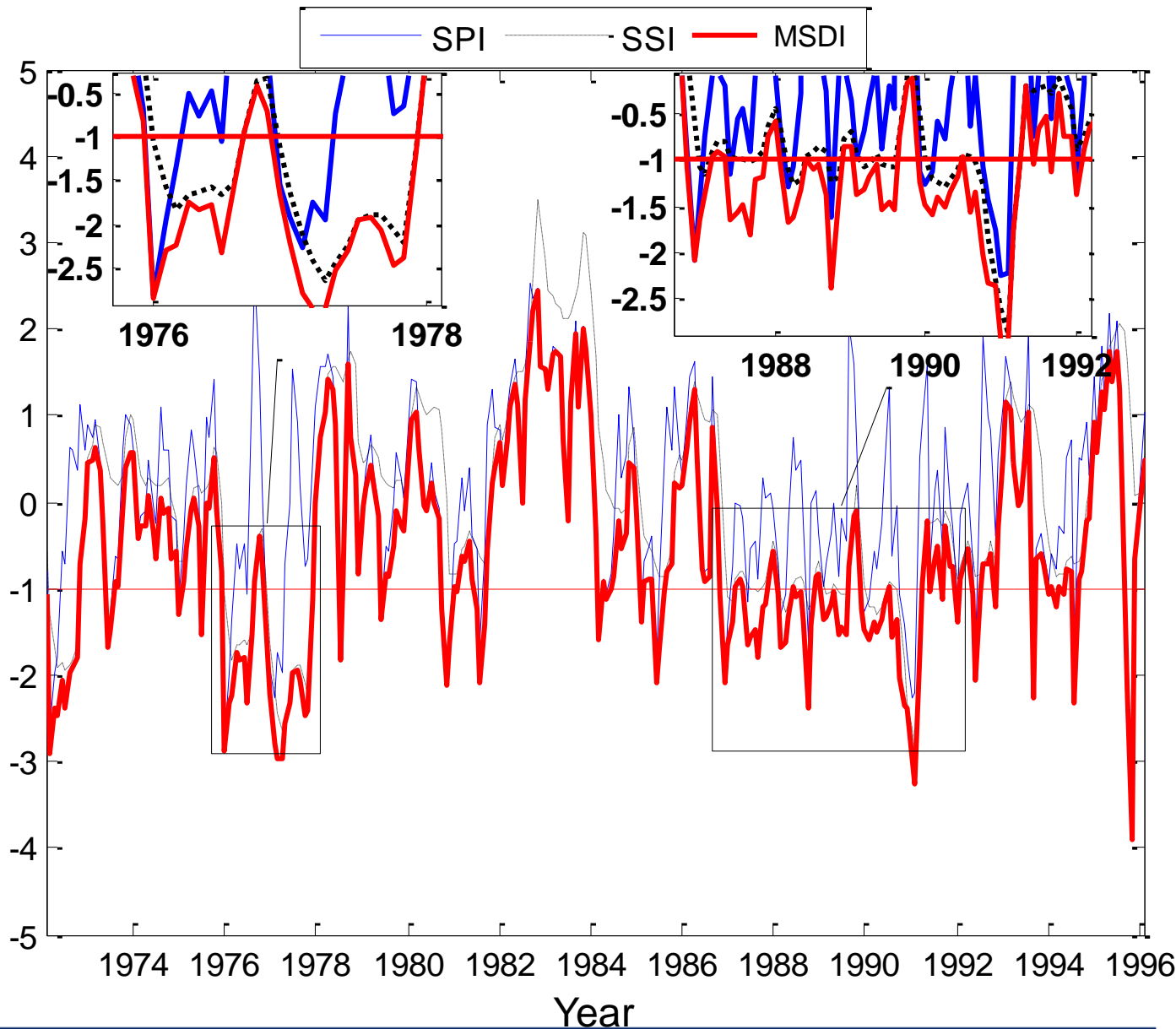


# Multi-Index Drought Monitoring

Monthly precipitation and soil moisture from 1/1932 to 12/2010

Precipitation : NCDC  
Soil Moisture: CPC

3-month SPI, SSI, MSDI for Climate Division 5 in California.



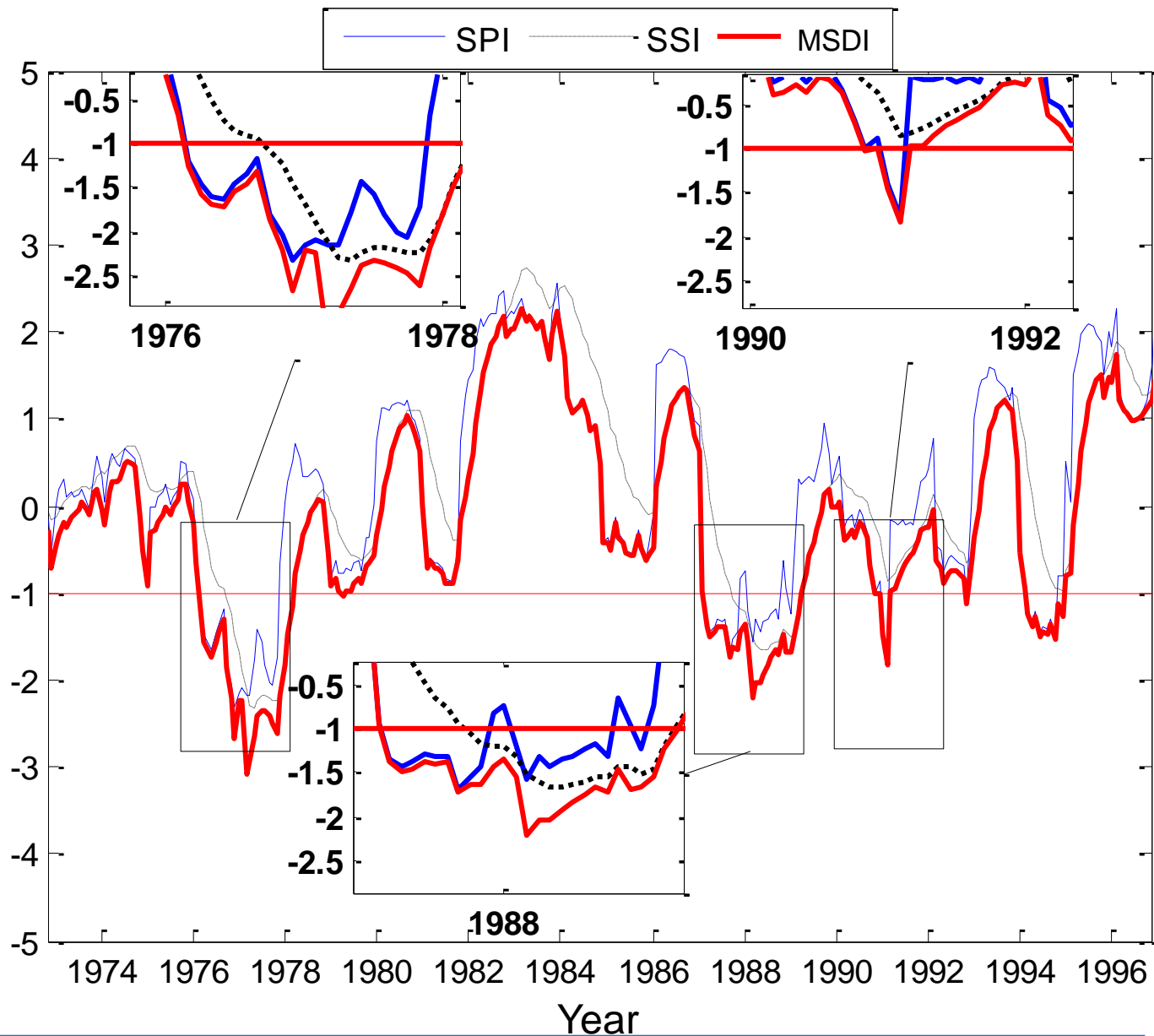


# Multi-Index Drought Monitoring

Monthly precipitation and soil moisture from 1/1932 to 12/2010

Precipitation : NCDC  
Soil Moisture: CPC

6-month SPI, SSI, MSDI for Climate Division 3 in California.

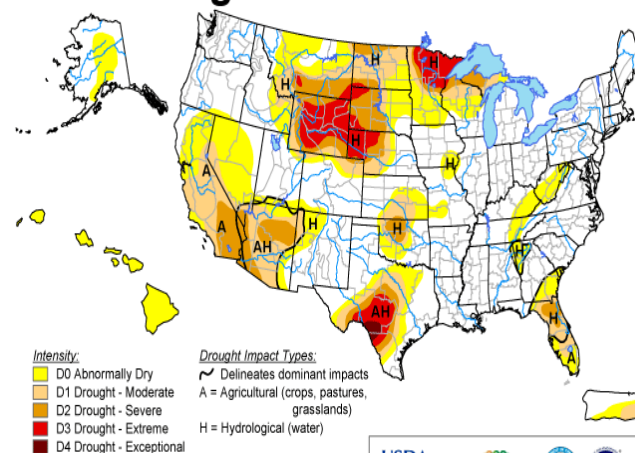




# Multi-Index Drought Monitoring

1-Month SPI and SSI Derived Using NASA MERRA-LAND Precipitation and soil moisture Data.

## U.S. Drought Monitor January 30, 2007 Valid 7 a.m. EST



The Drought Monitor focuses on broad-scale conditions. Local conditions may vary. See accompanying text summary for forecast statements.

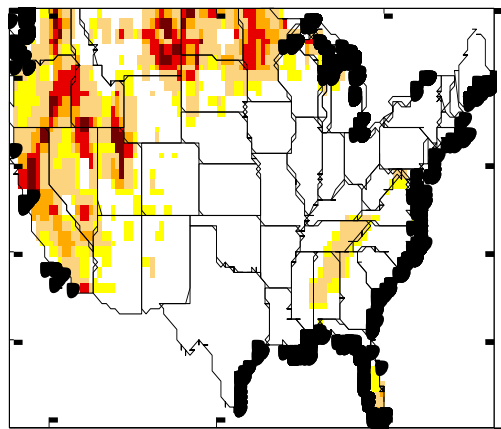
<http://drought.unl.edu/dm>



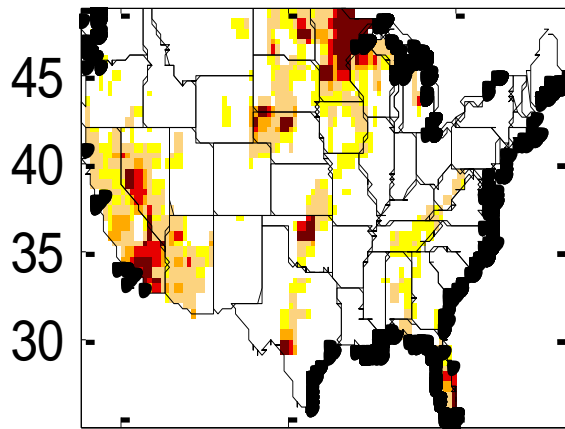
Released Thursday, February 1, 2007

Author: Brian Fuchs, National Drought Mitigation Center

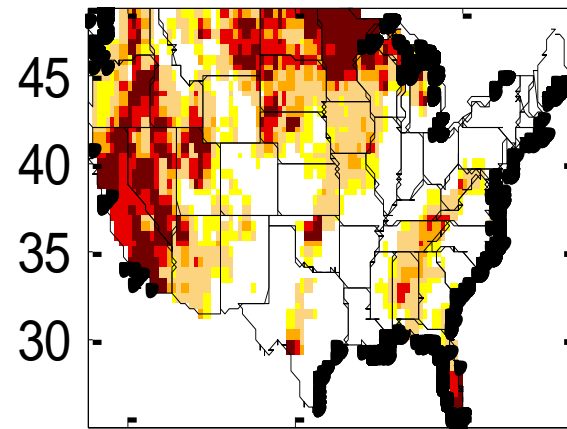
### SPI 2007-1



### SSI 2007-1



### MSDI 2007-1



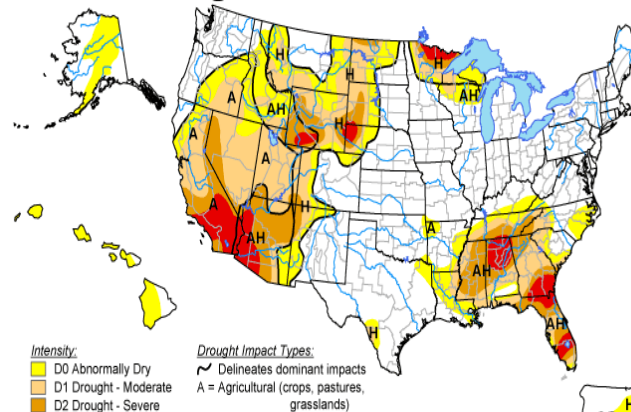


# Multi-Index Drought Monitoring

1-Month SPI and SSI Derived Using NASA MERRA-LAND Precipitation and soil moisture Data.

## U.S. Drought Monitor

May 1, 2007  
Valid 8 a.m. EDT



**Intensity:**

- D0 Abnormally Dry
- D1 Drought - Moderate
- D2 Drought - Severe
- D3 Drought - Extreme
- D4 Drought - Exceptional

**Drought Impact Types:**

- Delineates dominant impacts
- A = Agricultural (crops, pastures, grasslands)
- H = Hydrological (water)

The Drought Monitor focuses on broad-scale conditions. Local conditions may vary. See accompanying text summary for forecast statements.

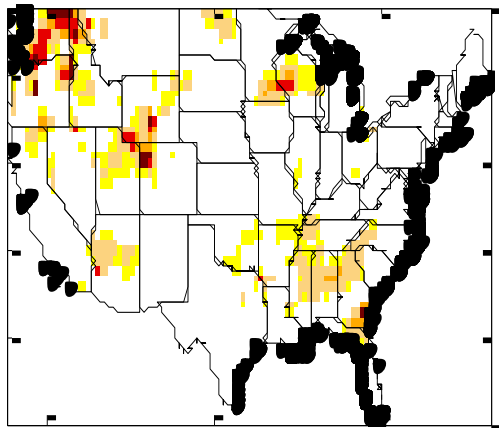


Released Thursday, May 3, 2007

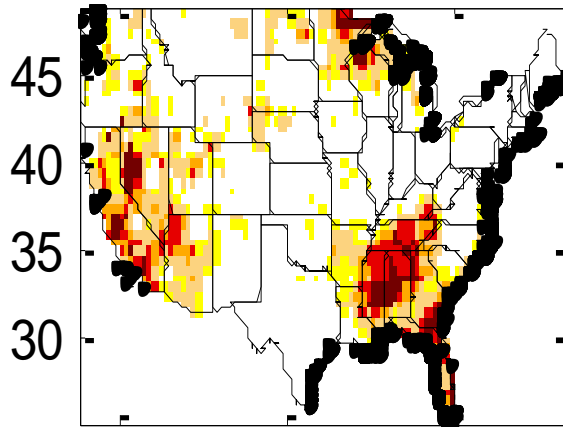
Author: Brian Fuchs, National Drought Mitigation Center

<http://drought.unl.edu/dm>

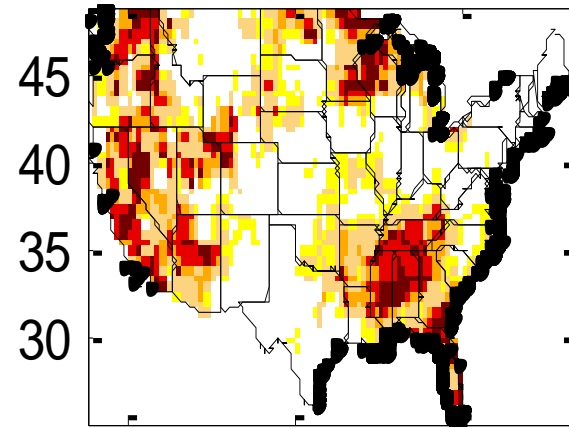
### SPI 2007-4



### SSI 2007-4



### MSDI 2007-4



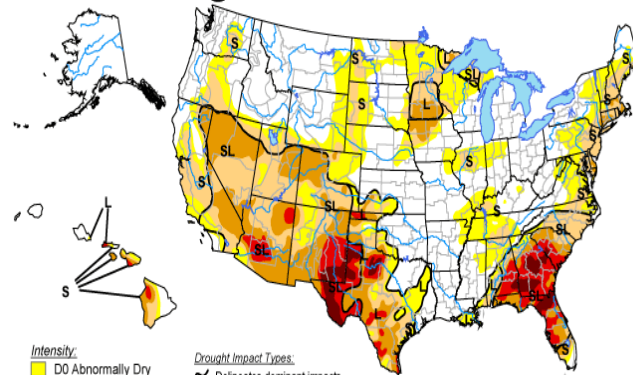


# Multi-Index Drought Monitoring

1-Month SPI and SSI Derived Using NASA MERRA-LAND Precipitation and soil moisture Data.

## U.S. Drought Monitor

May 1, 2012  
Valid 7 a.m. EDT



Intensity:

- D0 Abnormally Dry
- D1 Drought - Moderate
- D2 Drought - Severe
- D3 Drought - Extreme
- D4 Drought - Exceptional

Drought Impact Types:

- ~ Delineates dominant impacts
- S = Short-Term, typically <6 months (e.g. agriculture, grasslands)
- L = Long-Term, typically >6 months (e.g. hydrology, ecology)

The Drought Monitor focuses on broad-scale conditions. Local conditions may vary. See accompanying text summary for forecast statements.

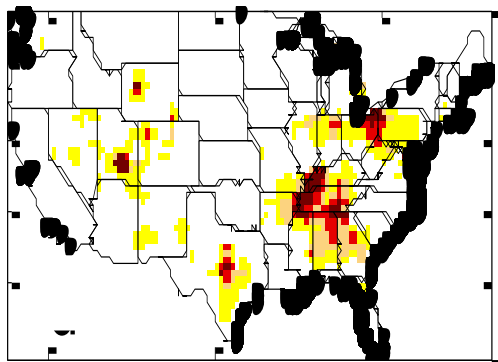


Released Thursday, May 3, 2012

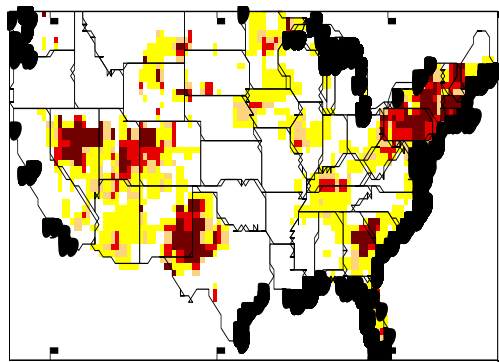
Author: Matthew Rosenkrans, NOAA/NWS/NCEP/CPC

<http://droughtmonitor.unl.edu/>

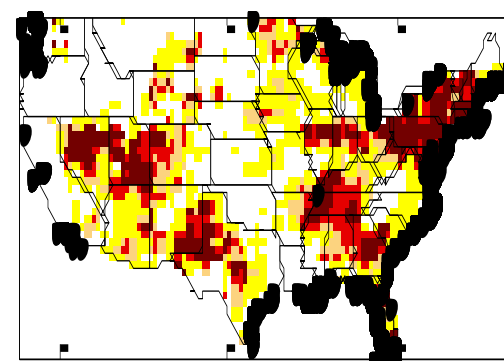
### SPI 2012-4



### SSI 2012-4



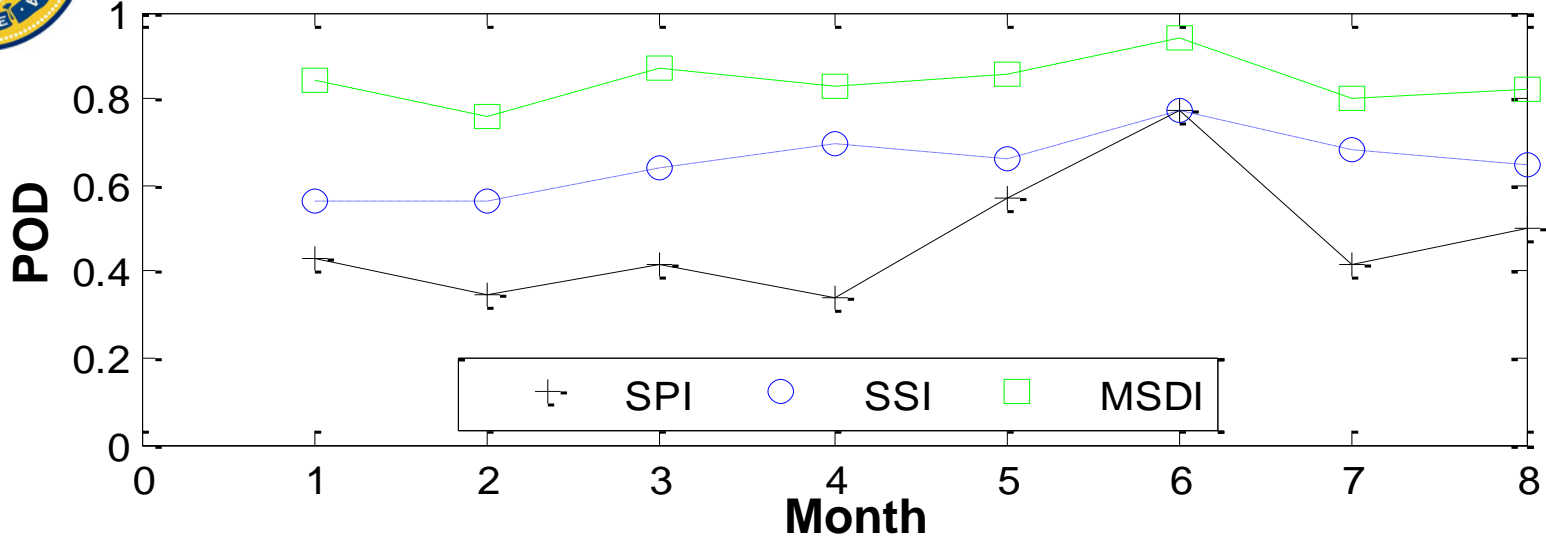
### MSDI 2012-4



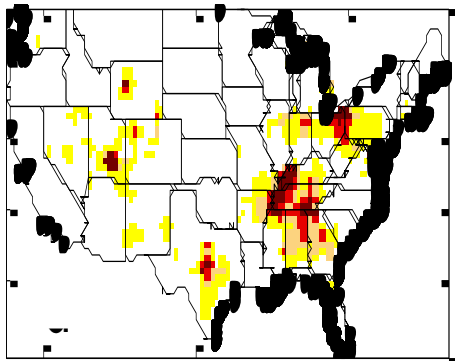


# Multi-Index Drought Monitoring

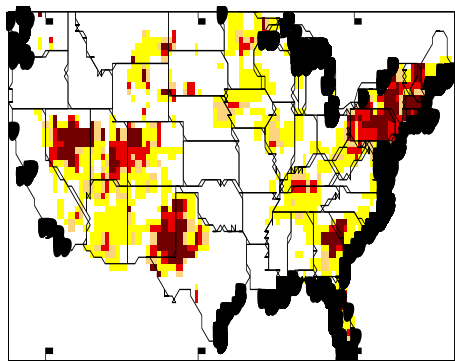
Probability of detection (POD) of drought using 1-month SPI, SSI and MSDI



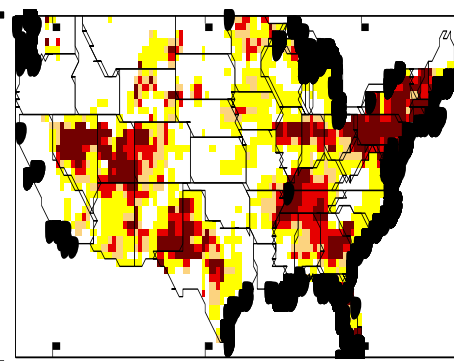
SPI 2012-04



SSI 2012-04

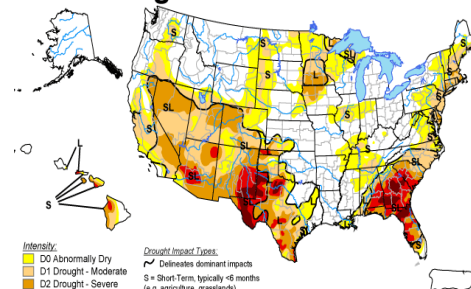


MSDI 2012-04



U.S. Drought Monitor

May 1, 2012  
Valid 7 a.m. EDT



**Intensity:**  
 D0 Abnormally Dry  
 D1 Drought - Moderate  
 D2 Drought - Severe  
 D3 Drought - Extreme  
 D4 Drought - Exceptional

**Drought Impact Types:**  
 ~ Delineates dominant impacts  
 S = Short-Term, typically <6 months (e.g. agriculture, grasslands)  
 L = Long-Term, typically >6 months (e.g. hydrology, ecology)

The Drought Monitor focuses on broad-scale conditions. Local conditions may vary. See accompanying text summary for forecast statements.



Released Thursday, May 3, 2012  
 Author: Matthew Rosenzans, NOAA/NWS/NCEP/PCPC

<http://droughtmonitor.unl.edu/>

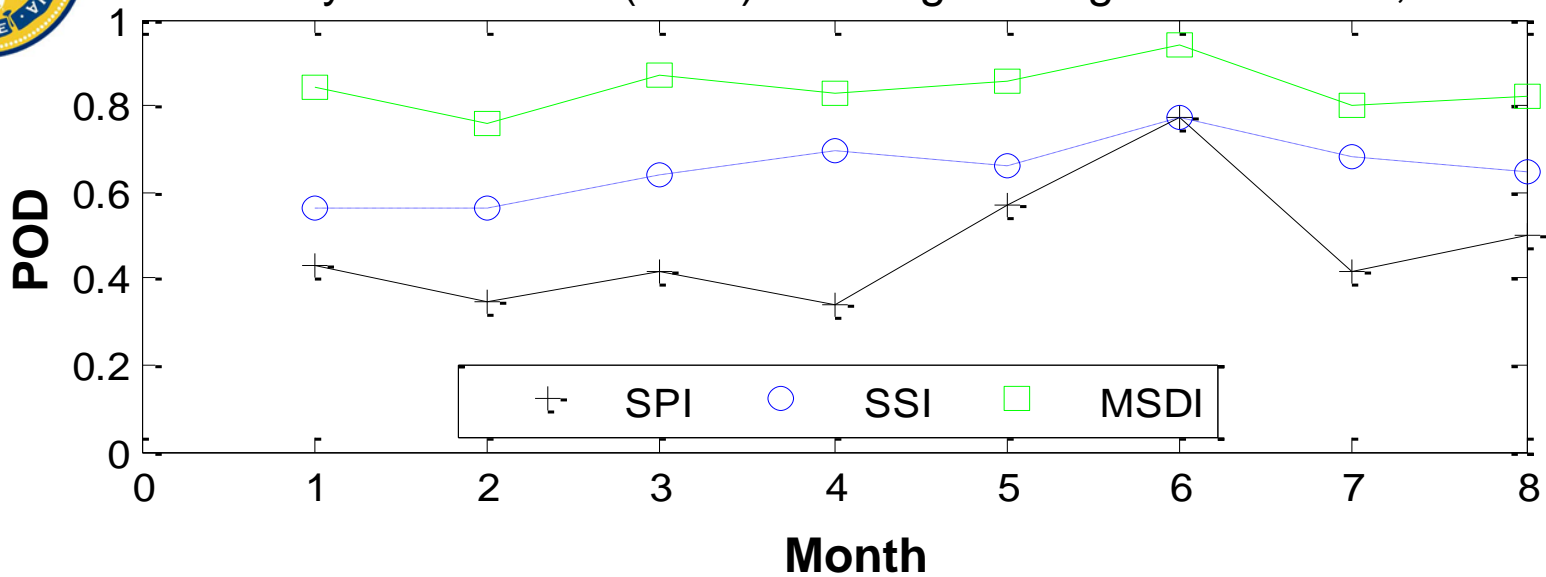




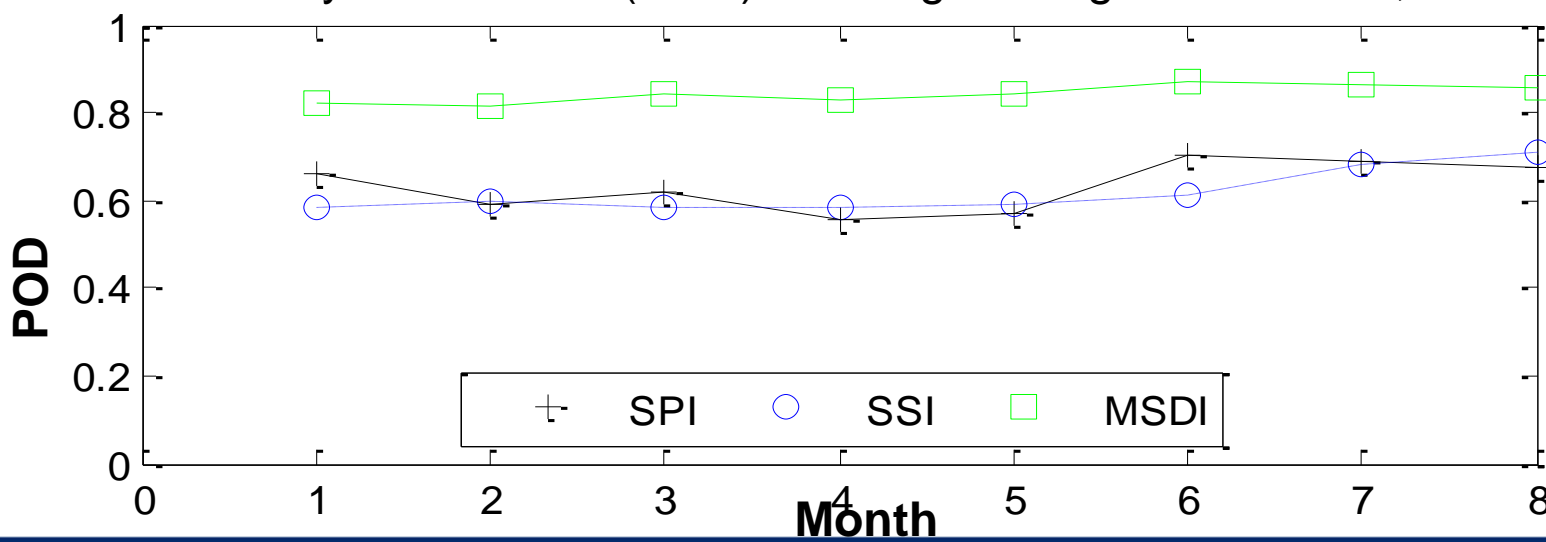


# Multi-Index Drought Monitoring

## Probability of detection (POD) of drought using 1-month SPI, SSI and MSDI



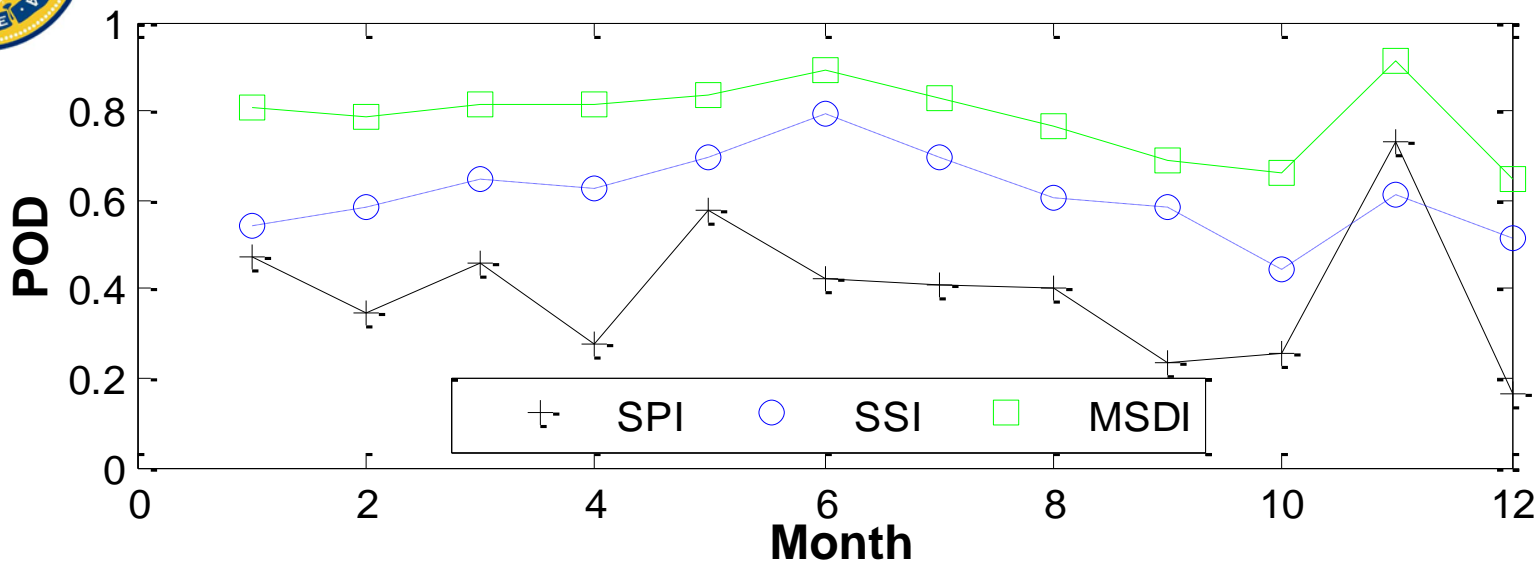
## Probability of detection (POD) of drought using 6-month SPI, SSI and MSDI





# Multi-Index Drought Monitoring

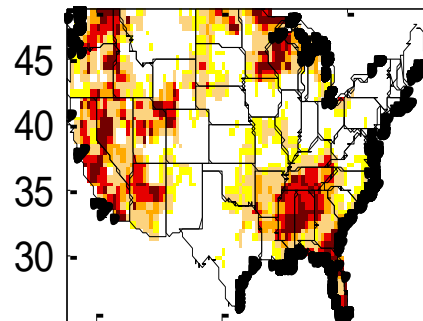
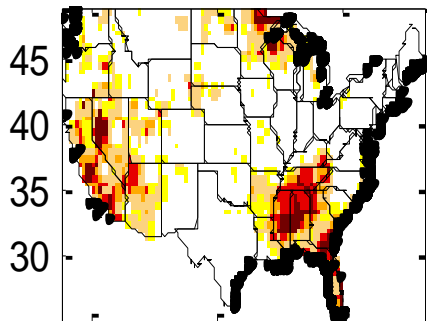
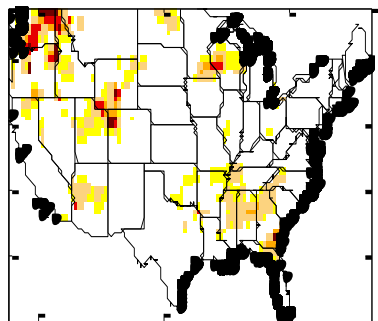
Probability of detection (POD) of drought using 1-month SPI, SSI and MSDI



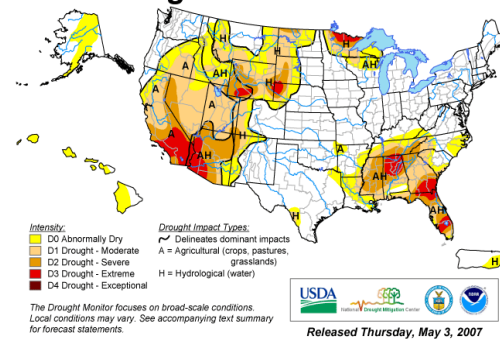
SPI 2007-4

SSI 2007-4

MSDI 2007-4



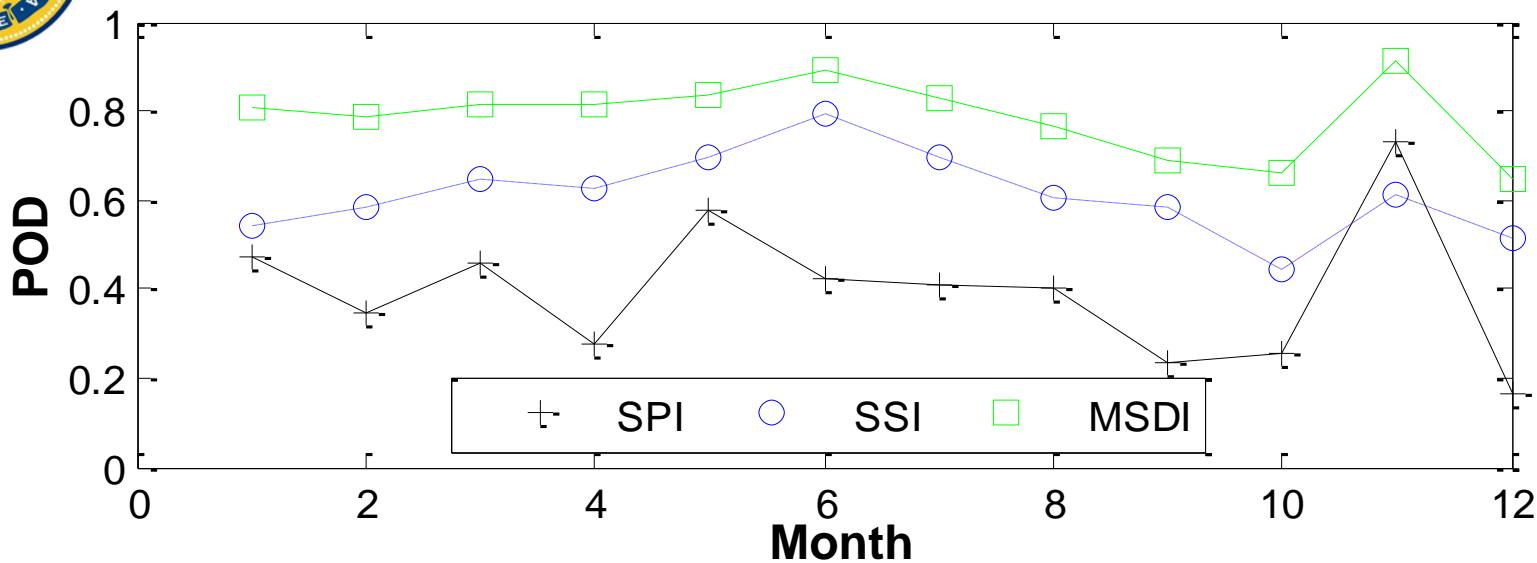
U.S. Drought Monitor May 1, 2007 Valid 6 a.m. EDT



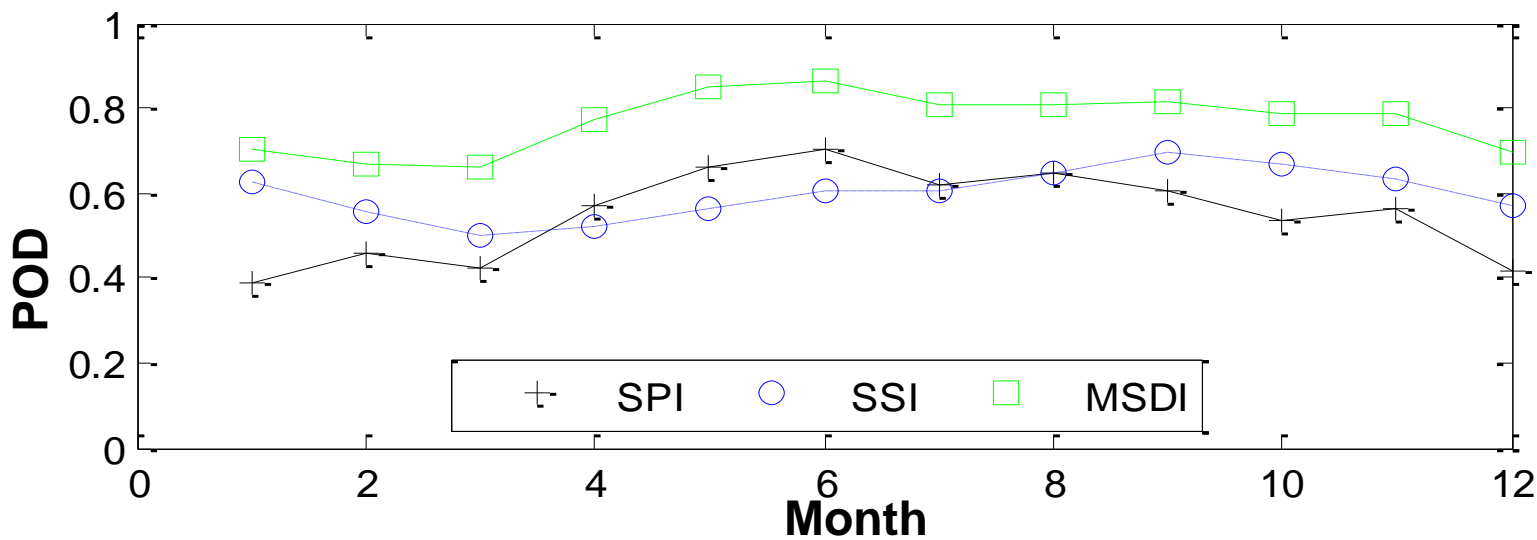


# Multi-Index Drought Monitoring

## Probability of detection (POD) of drought using 1-month SPI, SSI and MSDI



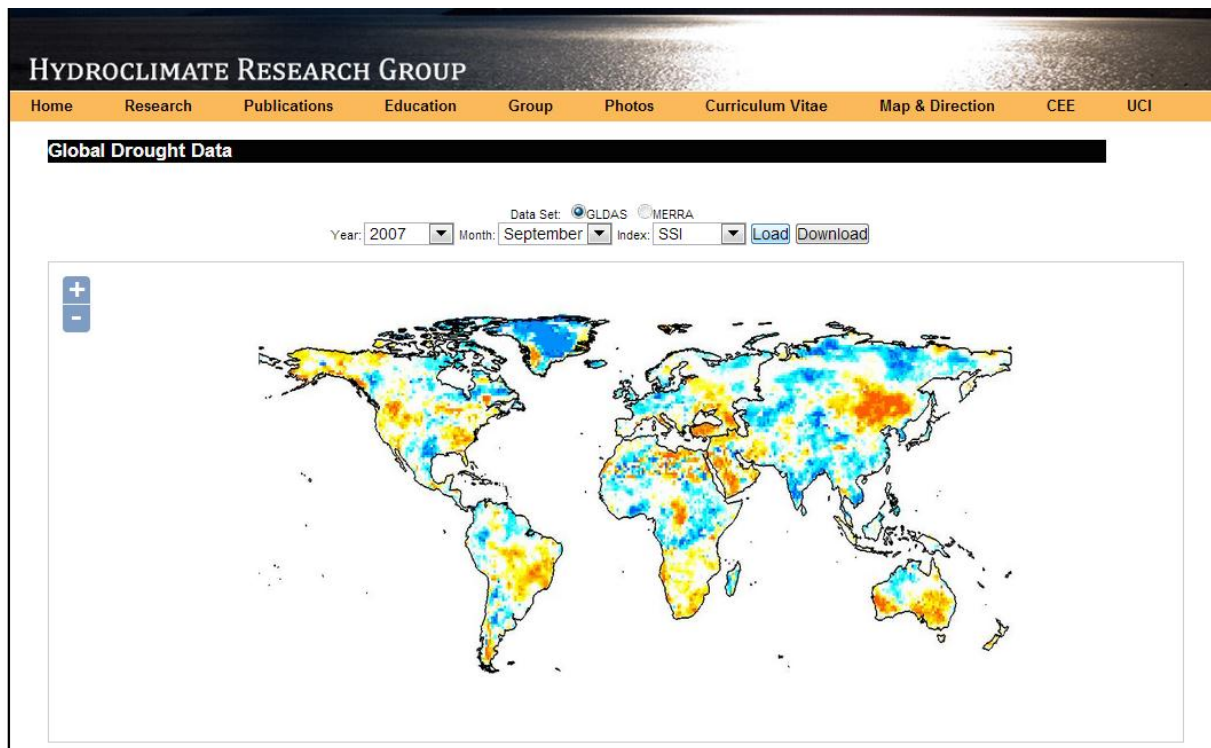
## Probability of detection (POD) of drought using 6-month SPI, SSI and MSDI





# Multi-Index Drought Monitoring

## Data Distribution System





# Multi-Index Drought Monitoring

## Data Distribution System

HYDROCLIMATE RESEARCH GROUP

Home Research Publications Education Group Photos Curriculum Vitae Map & Direction CEE UCI

Global Drought Data

Data Set:  GLDAS  MERRA  
Year: 2007 Month: September Index: SSI [Load](#) [Download](#)

Data Set:  GLDAS  MERRA  
Year: 2008 Month: April Index: SPI [Load](#) [Download](#)

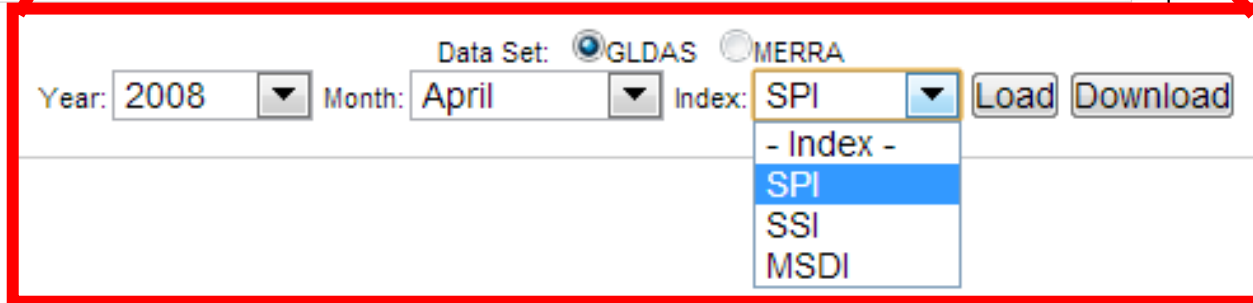
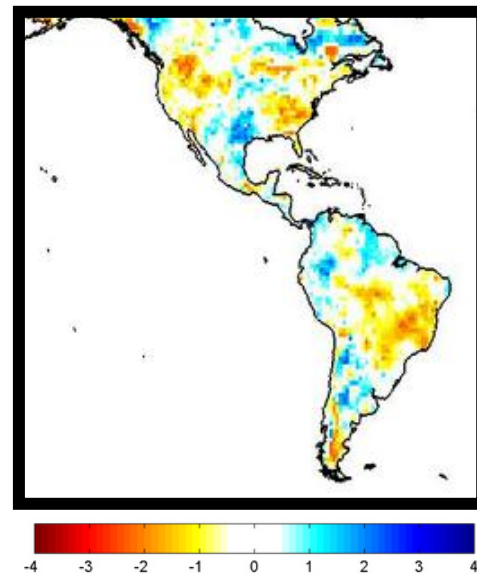
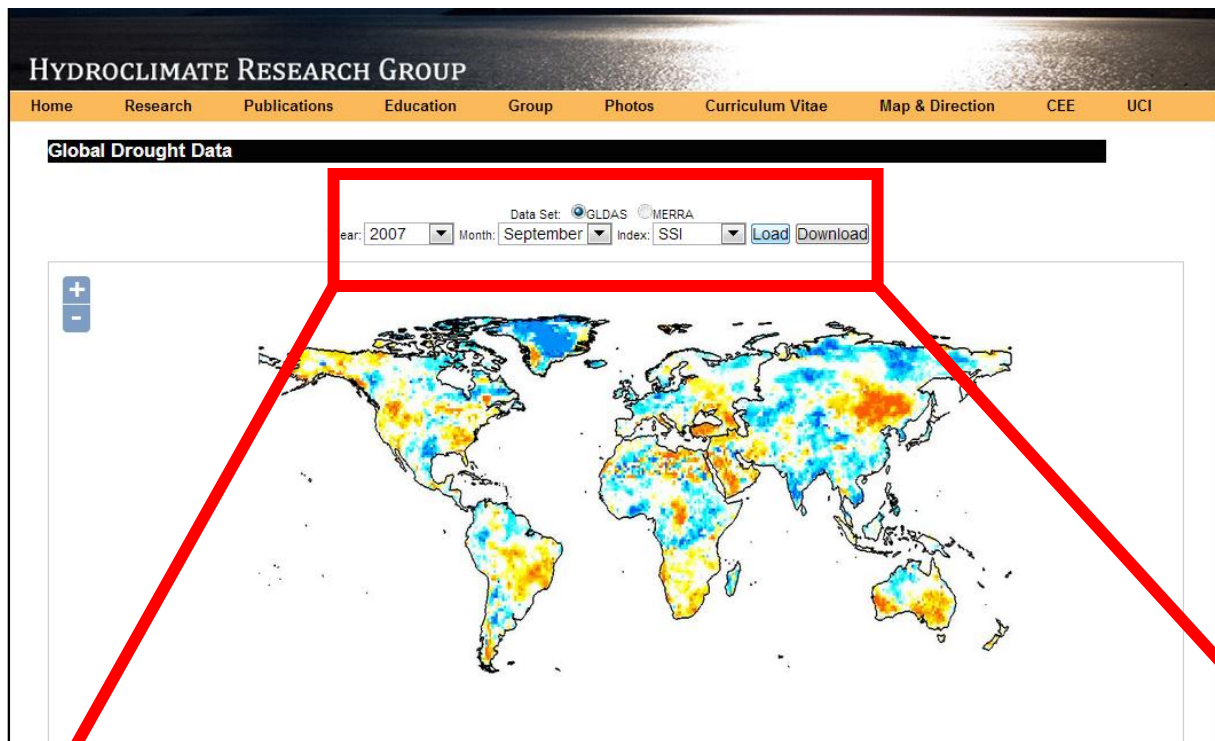
- Index -
- SPI
- SSI
- MSDI





# Multi-Index Drought Monitoring

## Data Distribution System





# Hydroclimate Research Group



**Zengchao Hao**



**Linyin Cheng**



**Ali Mehran**



**Lisa Damberg**



**Zhu Liu**



**Alireza Farahmand**



**Navid Nakhjiri**



**Sina Khosravi**

