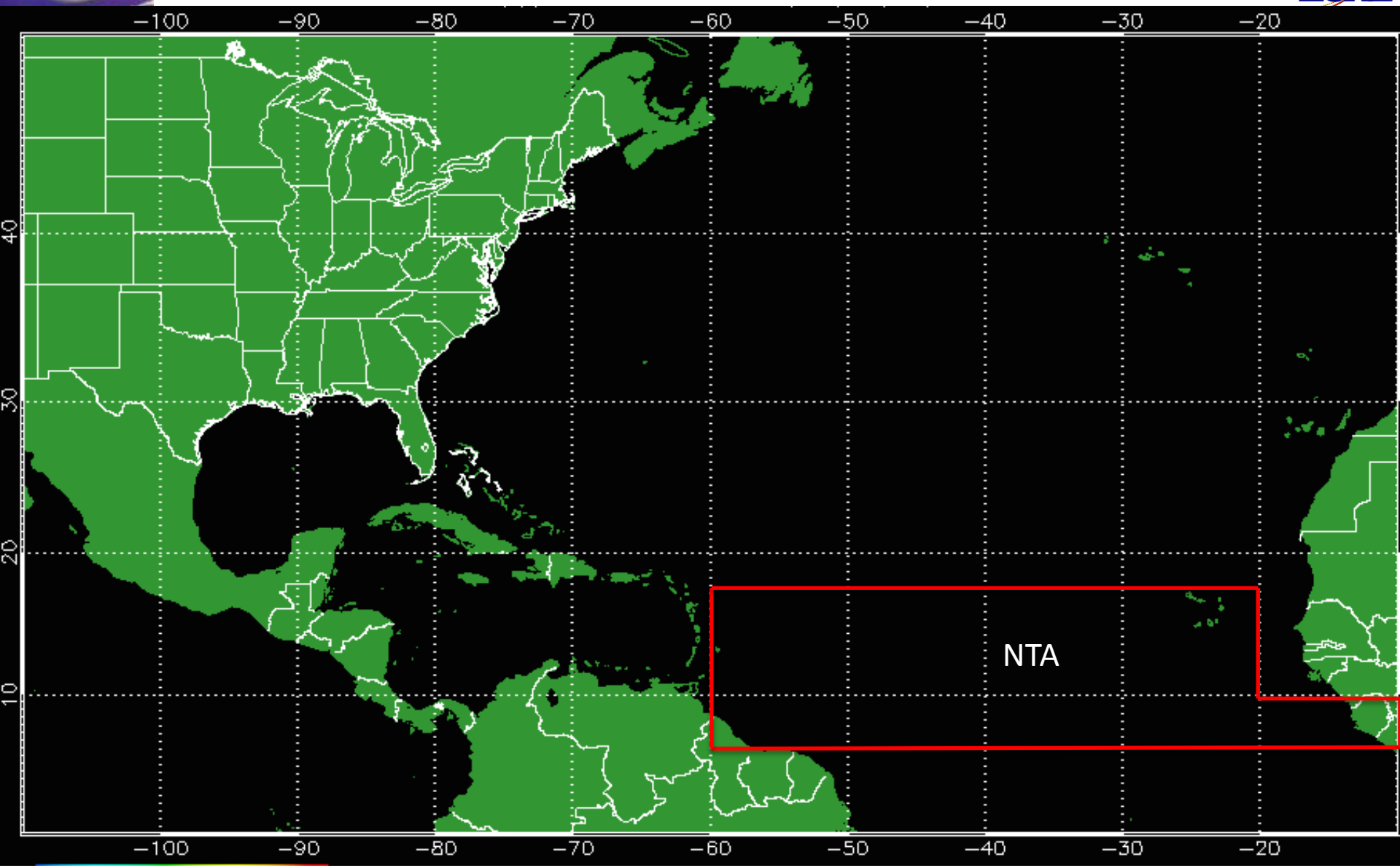
Stochastic forcing of north tropical Atlantic sea surface temperatures by the North Atlantic Oscillation

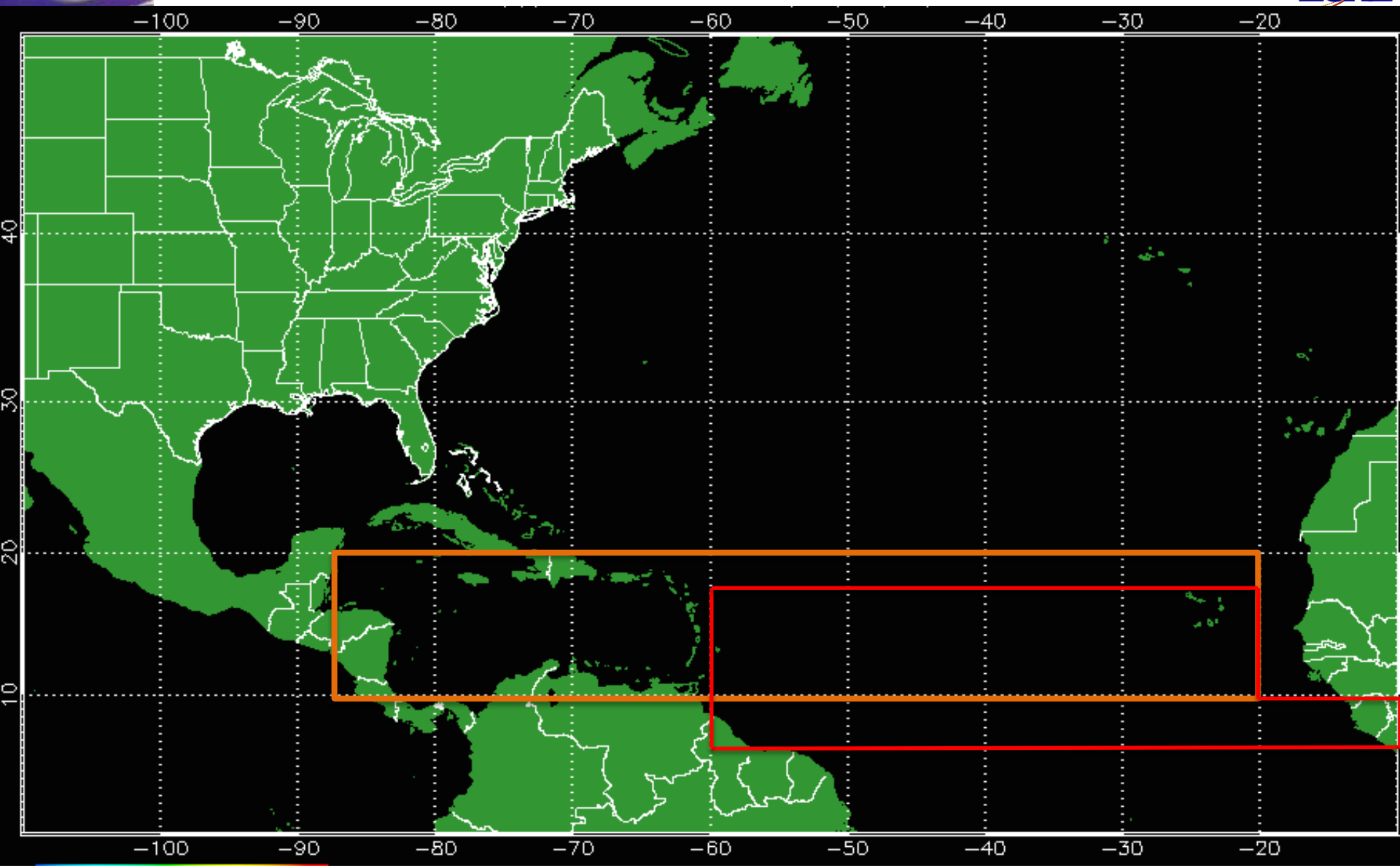
Cécile Penland[1] and Leslie Hartten[2,1]

[1]*NOAA/ESRL/Physical Sciences Division*

[2]*CIRES/University of Colorado*

Boulder, Colorado


−100
−90
−80
−70
−60
−50
−40
−30
−20
40
30
20
10
NTA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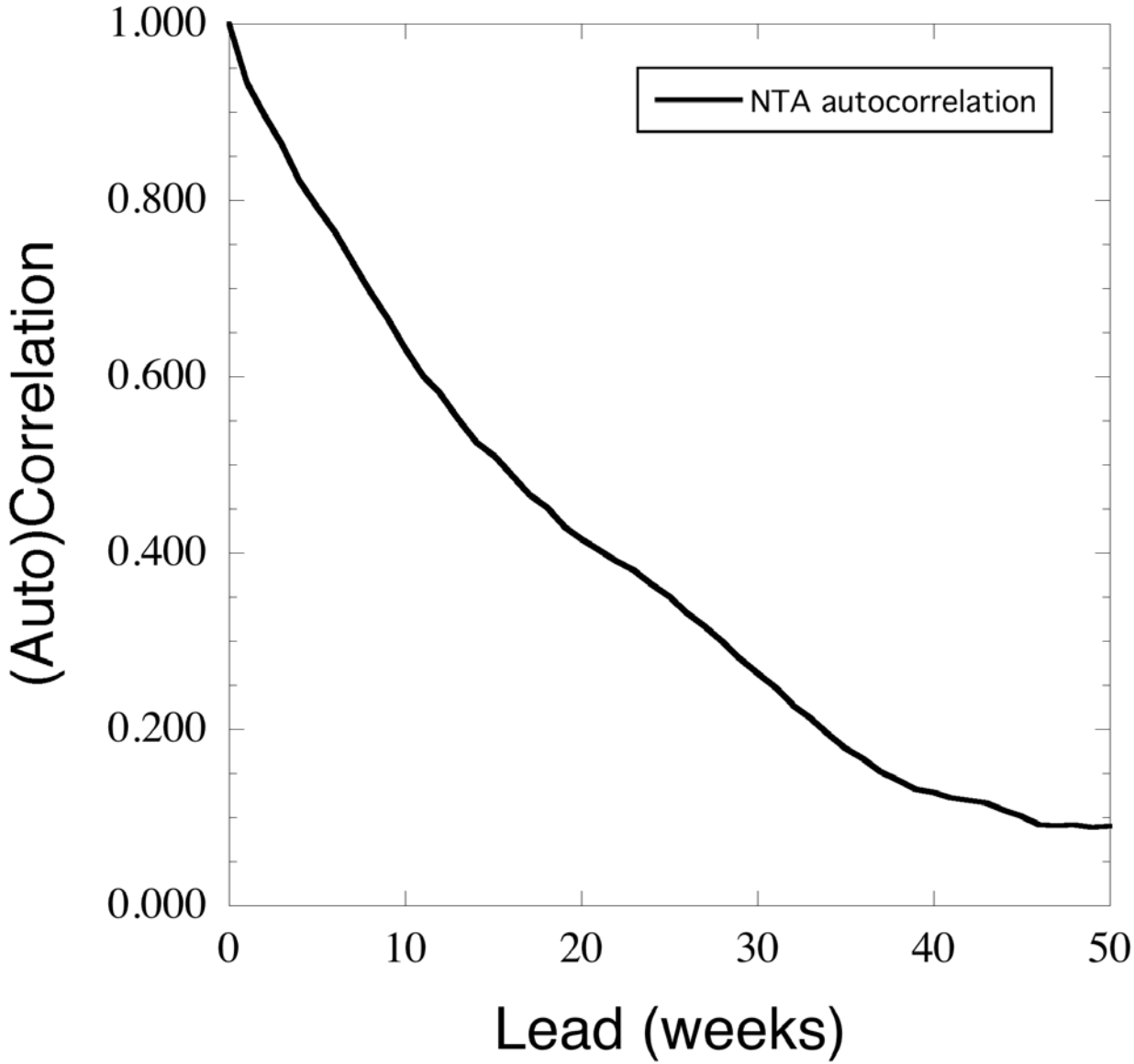
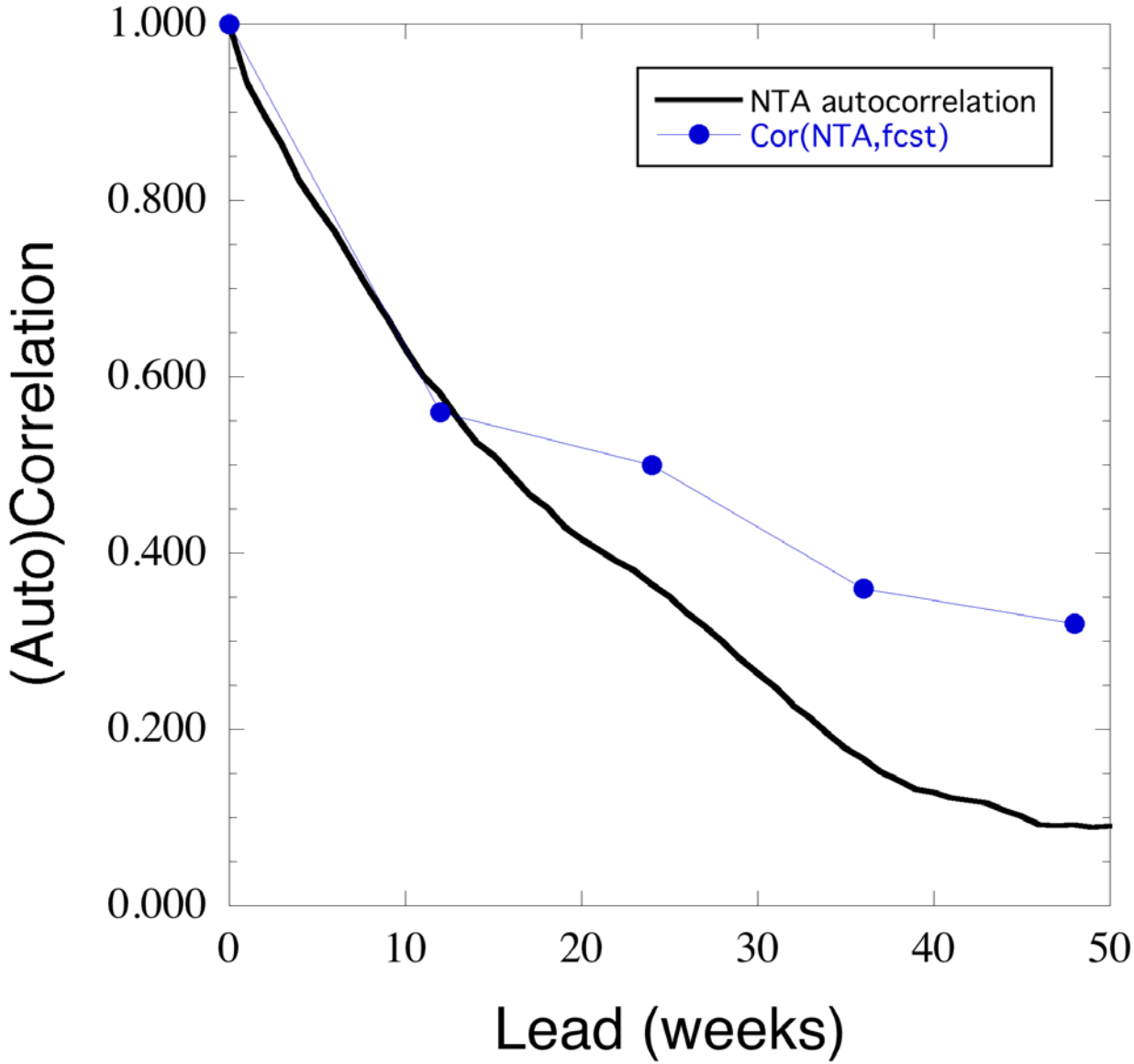
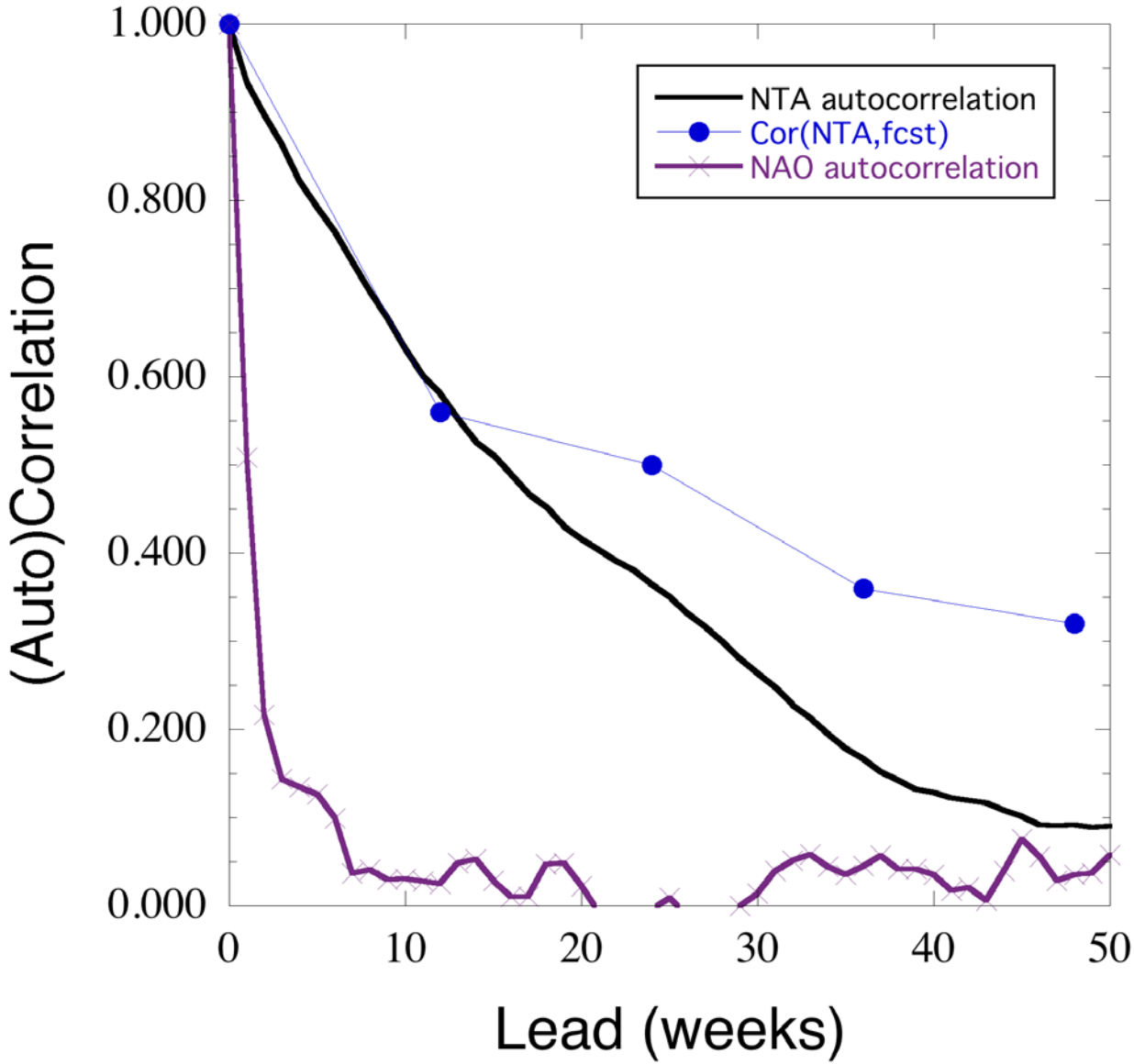
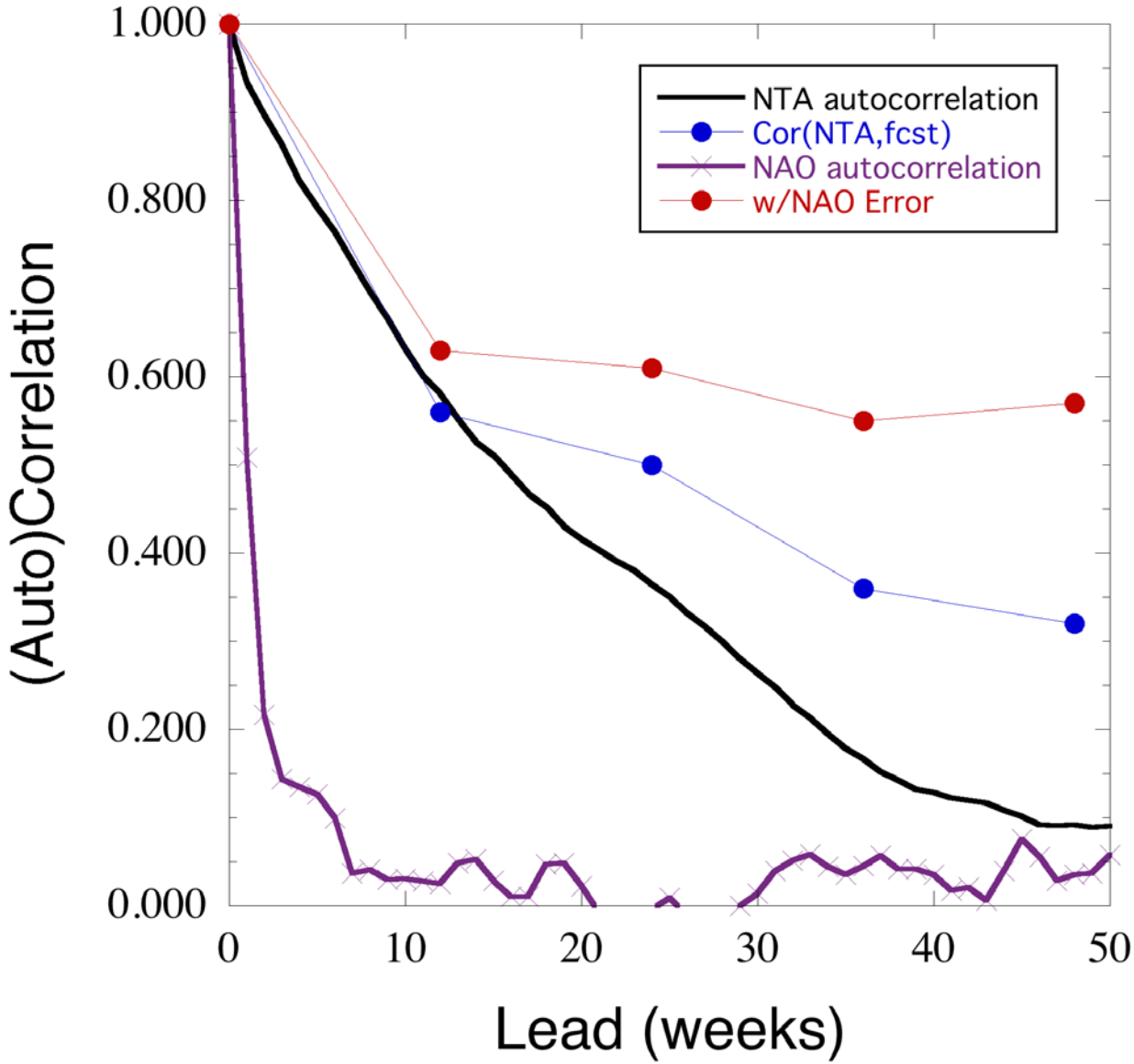

NTA autocorrelation
(Auto)Correlation
1.000
0.800
0.600
0.400
0.200
0.000
0
10
20
30
40
50
Lead (weeks)


NTA autocorrelation
Cor(NTA,fcst)
(Auto)Correlation
1.000
0.800
0.600
0.400
0.200
0.000
0
10
20
30
40
50
Lead (weeks)


NTA autocorrelation
Cor(NTA,fcst)
NAO autocorrelation
(Auto)Correlation
1.000
0.800
0.600
0.400
0.200
0.000
0
10
20
30
40
50
Lead (weeks)


NTA autocorrelation
Cor(NTA,fcst)
NAO autocorrelation
w/NAO Error
(Auto)Correlation
1.000
0.800
0.600
0.400
0.200
0.000
0
10
20
30
40
50
Lead (weeks)

Now, how did we get this?

Dynamical model:

$$\frac{d\boldsymbol{x}}{dt} = \mathbf{L}\boldsymbol{x} + \boldsymbol{N}(\boldsymbol{x},t)$$

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If $\boldsymbol{N}(t)$ is rapidly varying (see NAO autocorrelation), we can use, say, seasonal data to estimate $\mathbf{L}$ (use LIM) and weekly data to estimate $\boldsymbol{N}(t)$ via a centered difference.

$\boldsymbol{x}(t)$: weekly SST in tropical strip.

$\boldsymbol{X}(t)$: 3-month running mean SST in tropical strip.

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Estimate $\mathbf{L}$:

$$\exp(\mathbf{L}\tau) = <\boldsymbol{X}(t+\tau)\boldsymbol{X}^T(t)> [<\boldsymbol{X}(t)\boldsymbol{X}^T(t)>]^{-1}$$
$$= \mathbf{C}(\tau)\ \mathbf{C}^{-1}(0)$$

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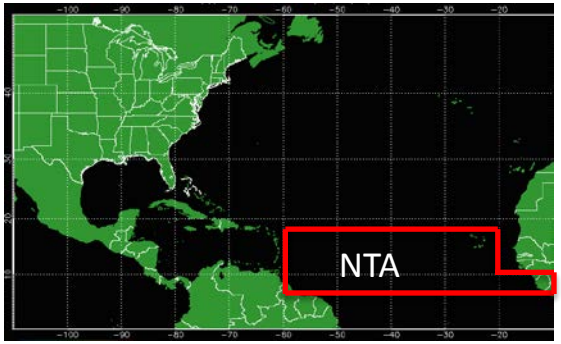
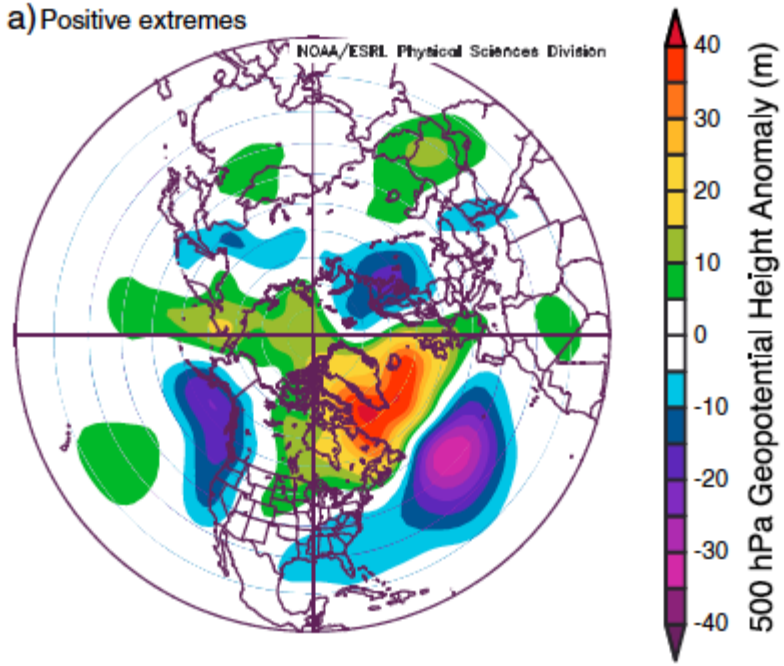
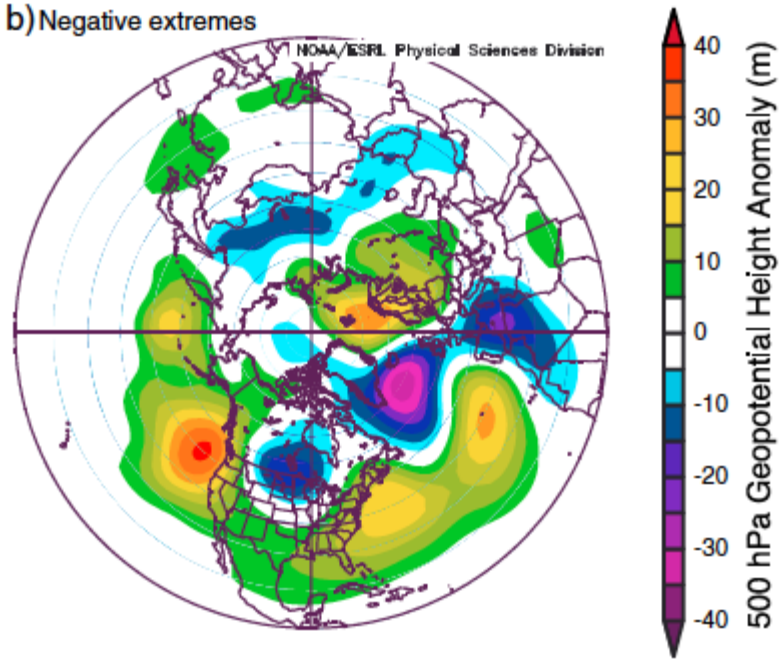
Estimate $\boldsymbol{N}(t)$:

$$\boxed{\boldsymbol{N}(t) \approx \frac{\boldsymbol{x}(t+\delta) - \boldsymbol{x}(t-\delta)}{2\delta} - \mathbf{L}\boldsymbol{x}(t)}$$

500mb height anomalies composited on +/- extremes of N_{NTA}.







Now we have an estimate of the stochastic forcing vector $\boldsymbol{N}(t)$ and can regress it onto any candidate time series η to get a regression matrix (or vector, if η is a single index) R.

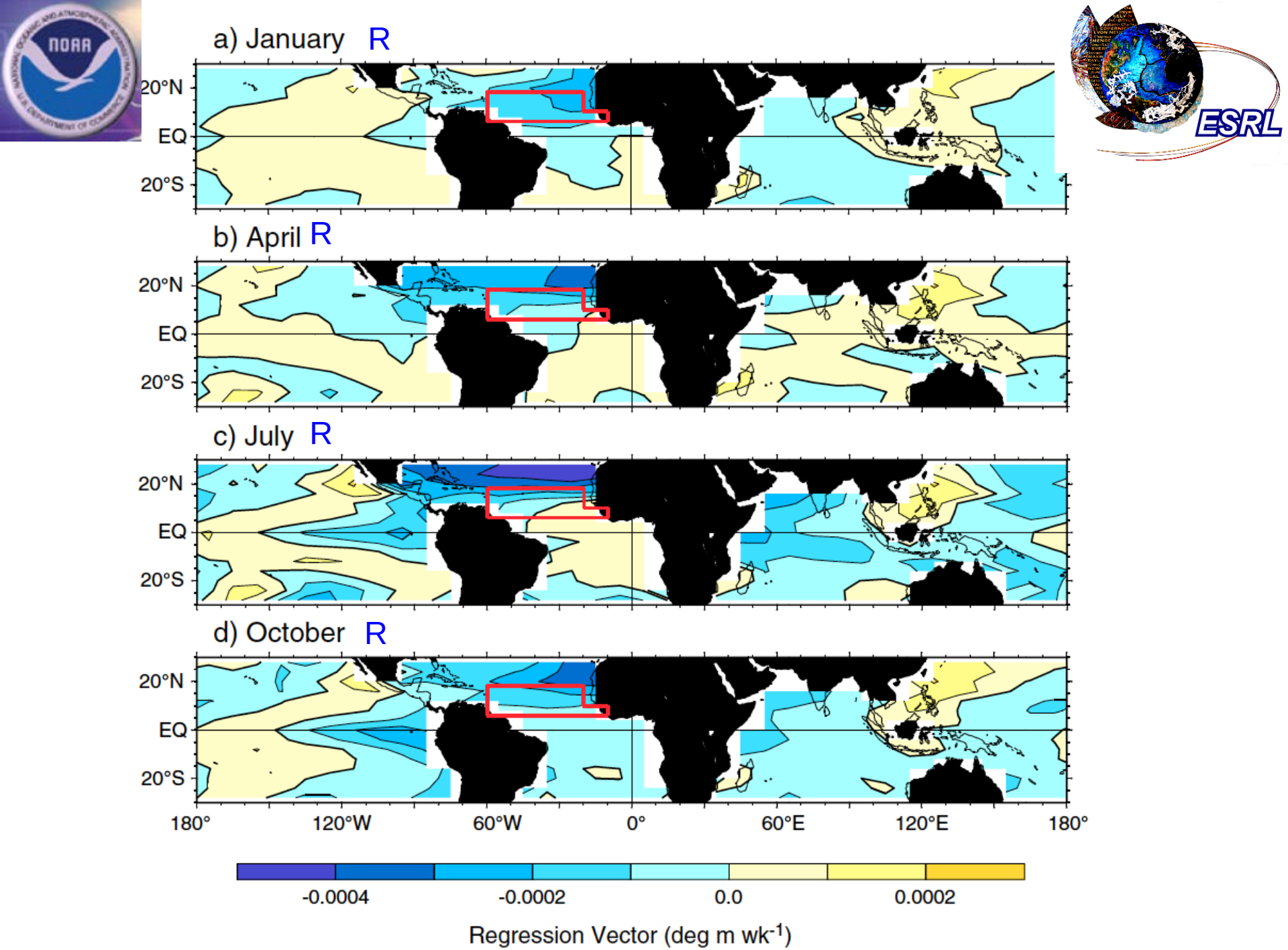
$$\boldsymbol{x}(t+\tau) = \exp(\mathbf{L}\tau)\boldsymbol{x}(t) + \int_0^{\tau} \exp(\mathbf{L}[\tau - t']) \, \mathrm{R}\eta(t+t')dt'$$

For $\eta(t)$, use NAO index from

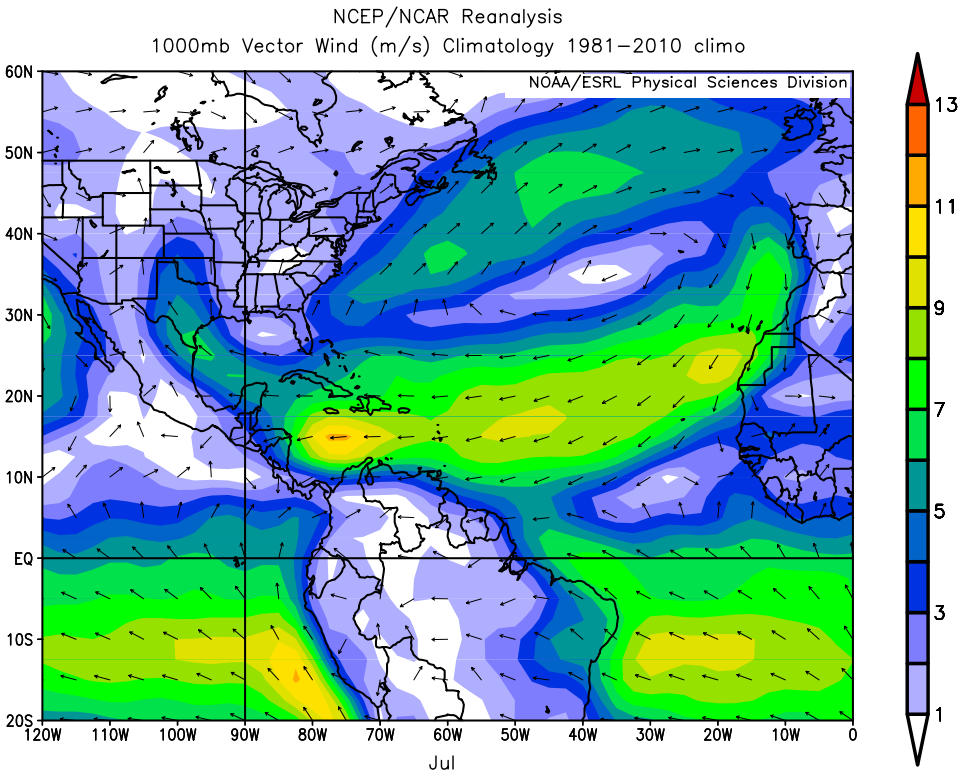
http://www.esrl.noaa.gov/psd/data/timeseries/daily/NAO/

Calculation Method:

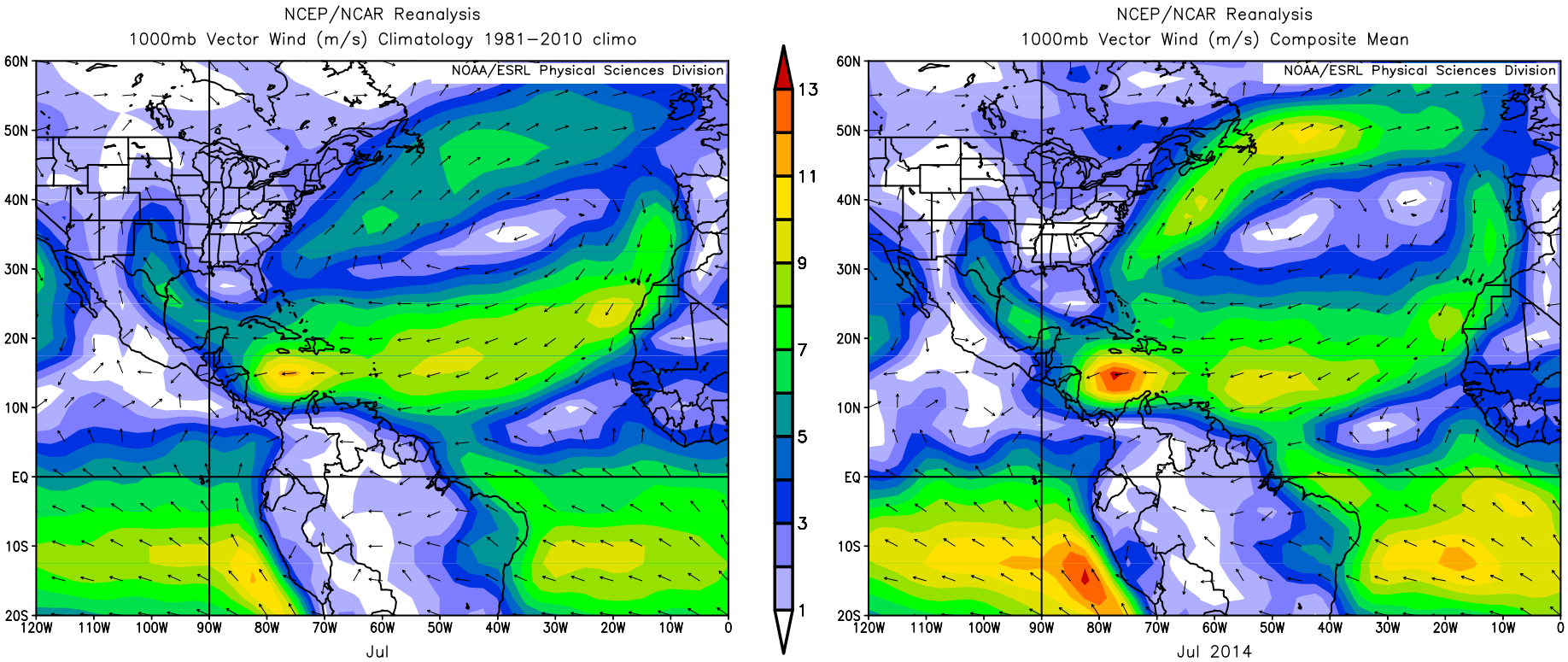
The indices are based on centers-of-action of 500mb height patterns. These time series utilize the NCEP/NCAR R1 dataset. The area averaged region 55-70N;70W-10W is subtracted from 35-45N; 70W-10W. The 1981-2010 period was used as climatology. Before computing these indices the 500mb height fields are spectrally truncated to total wavenumber 10 in order to emphasize large-scale aspects of the teleconnections.


a) January R
b) April R
c) July R
d) October R
20°N
EQ
20°S
180°
120°W
60°W
0°
60°E
120°E
180°
-0.0004
-0.0002
0.0
0.0002
Regression Vector (deg m wk-1)

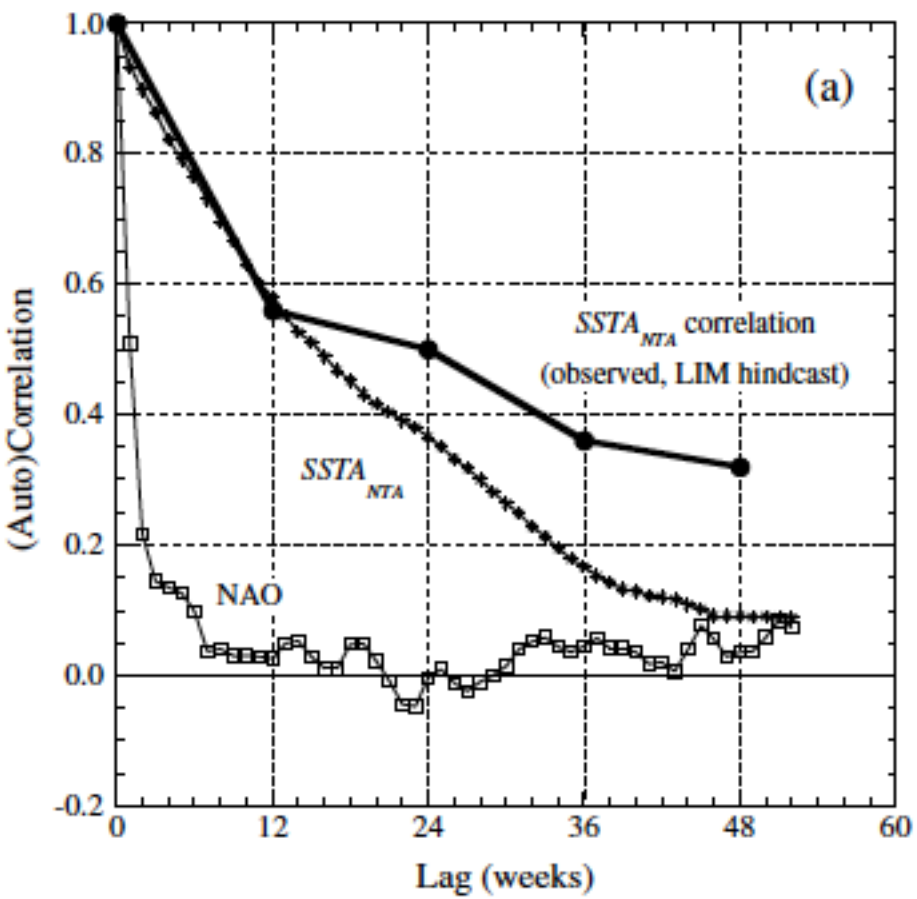
Digression:

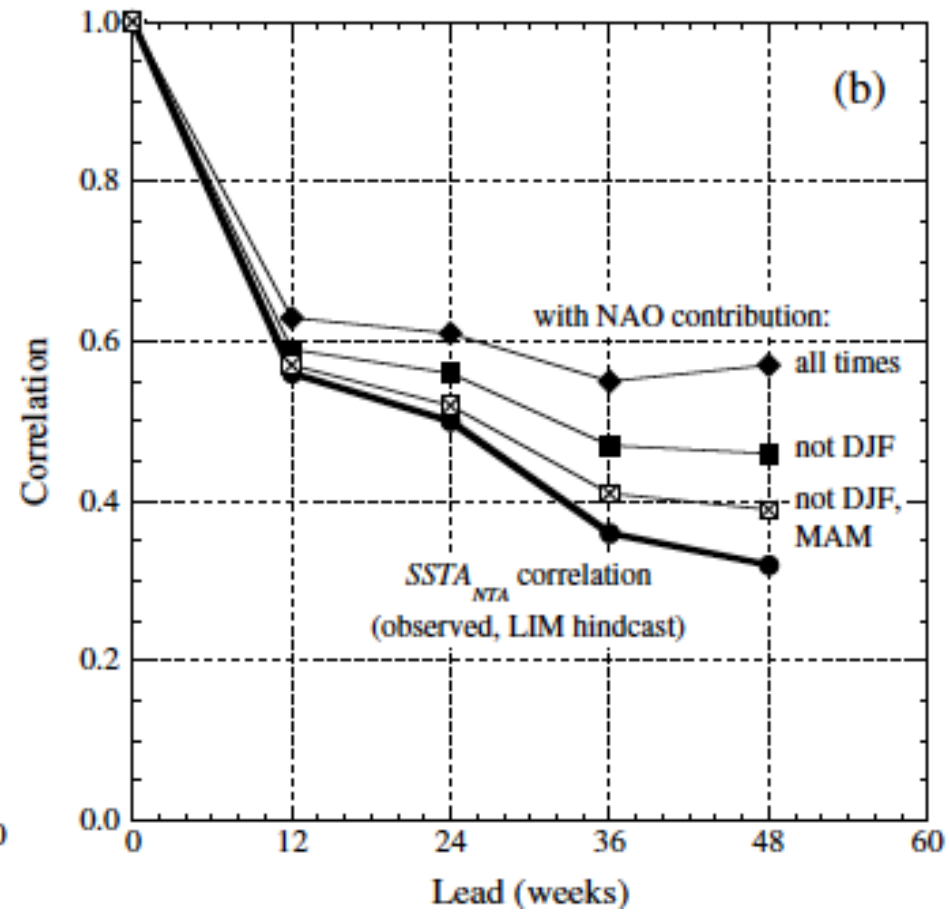

NCEP/NCAR Reanalysis
1000mb Vector Wind (m/s) Climatology 1981–2010 climo
NOAA/ESRL Physical Sciences Division
60N
50N
40N
30N
20N
10N
EQ
10S
20S
120W
110W
100W
90W
80W
70W
60W
50W
40W
30W
20W
10W
0
Jul
13
11
9
7
5
3
1

Digression (cont.)



$$\boldsymbol{x}(t+\tau) = \exp(\mathbf{L}\tau)\boldsymbol{x}(t) + \int_0^{\tau} \exp(\mathbf{L}[\tau - t']) \, R\eta(t+t') \, dt'$$





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- Most details in Penland and Hartten (2014) *GRL.*