Scaled SVD analysis and its application to tropical-extratropical teleconnections

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Outline

- ‘Scaling’ transformation
- Scaled SVD Analysis (SSVD) and relationship with other methods
- Synthetic results – appropriate scaling values
- Teleconnections between tropical precipitation & NH circulation – ENSO & ENSO Modoki

Scaling transformation

- Given a variable of anomalies that varies as space × time (matrix X)
- Compute EOFs:

$$X = \underbrace{\boxed{U_X}}_{\text{patterns}}\underbrace{\boxed{S_X}}_{\text{*amplitude*}}\underbrace{\boxed{V_X{}^T}}_{\text{time series}}$$

- *Scale* the data according to a fixed parameter α through a linear transformation

$$\boxed{X^* = \left(U_X S_X{}^{\alpha-1} U_X{}^T\right) X = U_X S_X{}^{\alpha} V_X{}^T}$$

- Acts to apply a *degree* of normalization (at each spatial point) & decorrelation (between differing spatial points)
  - α = 1: no change; α = 0: whitening (DelSole & Tippett 2007)
  - 0 < α < 1: 'partial whitening', term of Donohue et al. (2007) from acoustics (analogous use with spherical harmonics)

Scaled SVD Analysis

- Given two anomalous variables X and Y, compute EOFs and scale both datasets (depending on α and β)

$$X^* = U_X S_X{}^{\alpha} V_X{}^T \qquad Y^* = U_Y S_Y{}^{\beta} V_Y{}^T$$

- Cross-covariance of scaled data: $X^* Y^{*T}$
- Diagonalize with $SVD(X^* Y^{*T})$: singular vectors maximize generic measure of relation $\left(\hat{\sigma}_x{}^{\alpha} \hat{\rho}_{x,y} \hat{\sigma}_y{}^{\beta}\right)^2$
- Scaling as a prior constraint:
  - Specify role of amplitude
  - Intuitively more logical than EOF truncation (Barnett & Preisendorfer 1987)

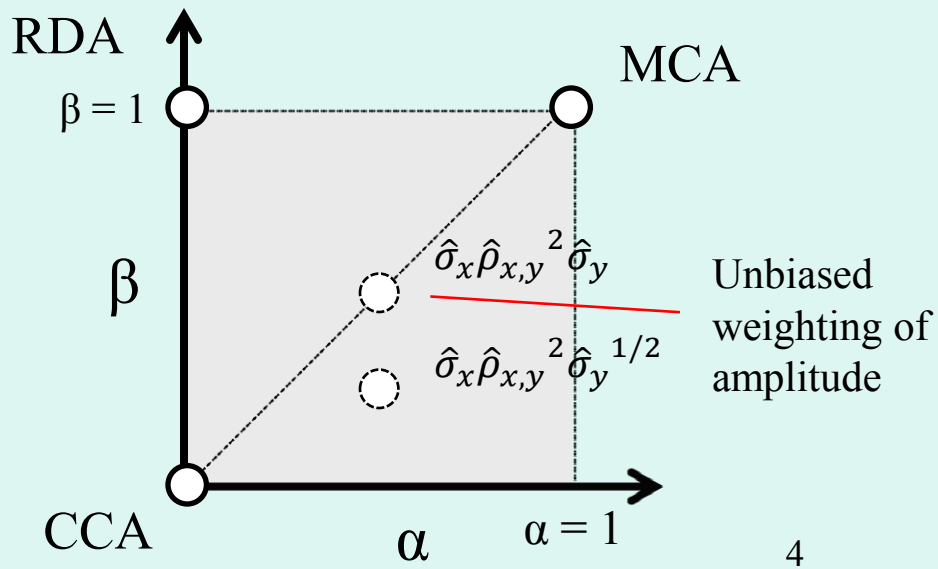
Unique solutions

- Tippett et al. (2008): basis from linear regression
  - Maximum Covariance Analysis (MCA), Canonical Correlation Analysis (CCA) & Redundancy Analysis (RDA)
- Recoverable as unique solutions of SSVD
  - α = 1 (0) ensures strict orthogonality in patterns (variates)

- MCA (α = β = 1) maximizes covariance^2 given by $\left(\hat{\sigma}_x \hat{\rho}_{x,y} \hat{\sigma}_y\right)^2$

- CCA (α = β = 0) maximizes correlation^2 given by $\hat{\rho}_{x,y}{}^2$

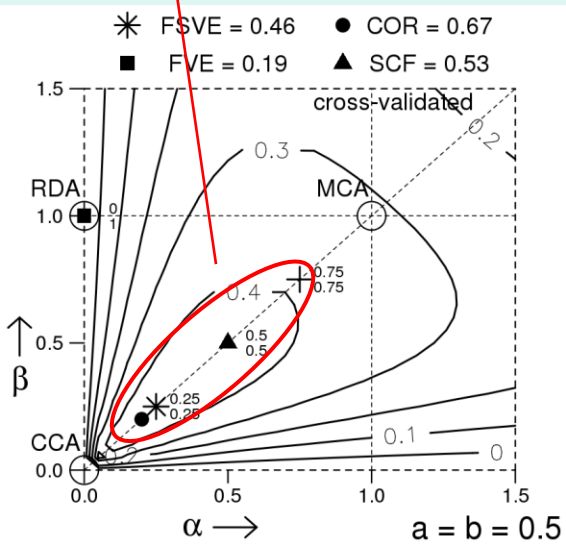
- RDA (α = 0, β = 1) maximizes variance of *Y* explained by *X* given by $\left(\hat{\rho}_{x,y} \hat{\sigma}_y\right)^2$

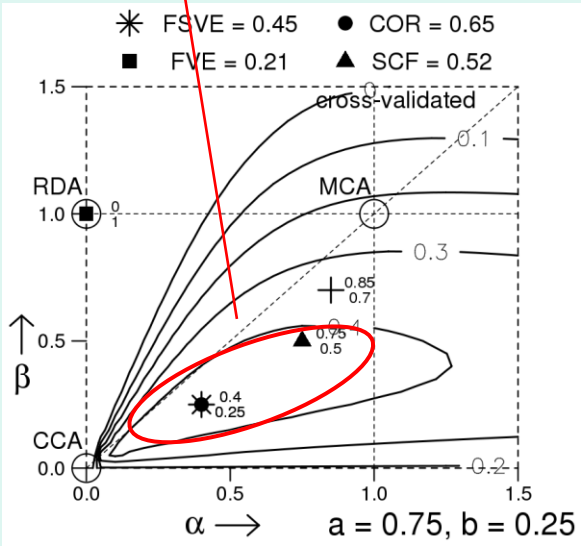


What scaling values to use?

- Known signal embedded in synthetic data:
  - intermediate solutions tend to isolate coupled signals better
  - α, β of highest skill proportional to true signal-to-noise ratio

Difference in solutions proportional to sampling





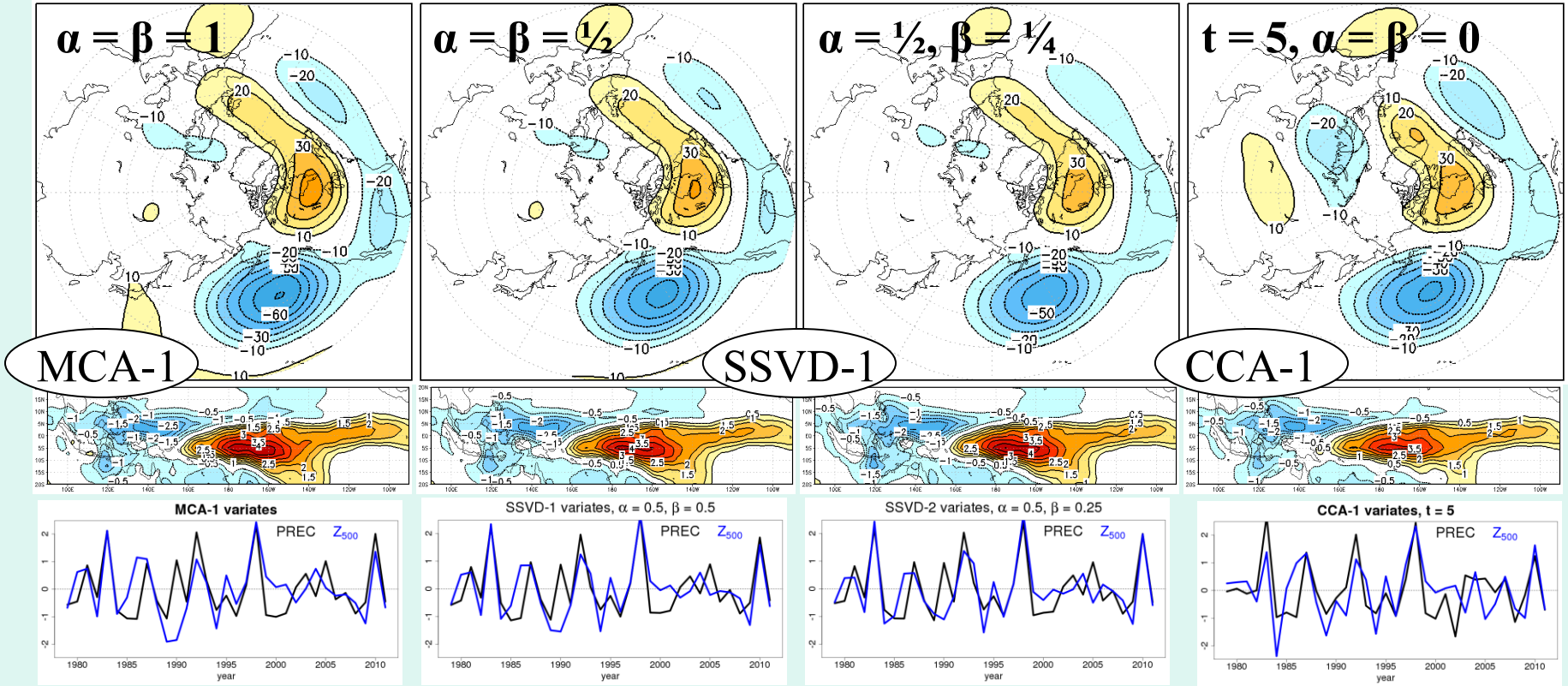
X has larger signal-to-noise ratio than *Y* → α > β produces better results

NH circulation ↔ tropical Pacific

- Relate wintertime tropical Pacific precipitation (Feb-Mar, CMAP) and NH 500 hPa geopotential height (Jan-Feb, NCEP2)
  - 1979-2011 (33 years)
  - remove mean + trend + quadratic term
- Contrast two leading modes of SSVD alongside those of MCA and CCA
- Compute correlation (*COR*) and squared covariance fraction (*SCF*) using leave-3-out cross-validation
- Prior expectation for teleconnections:

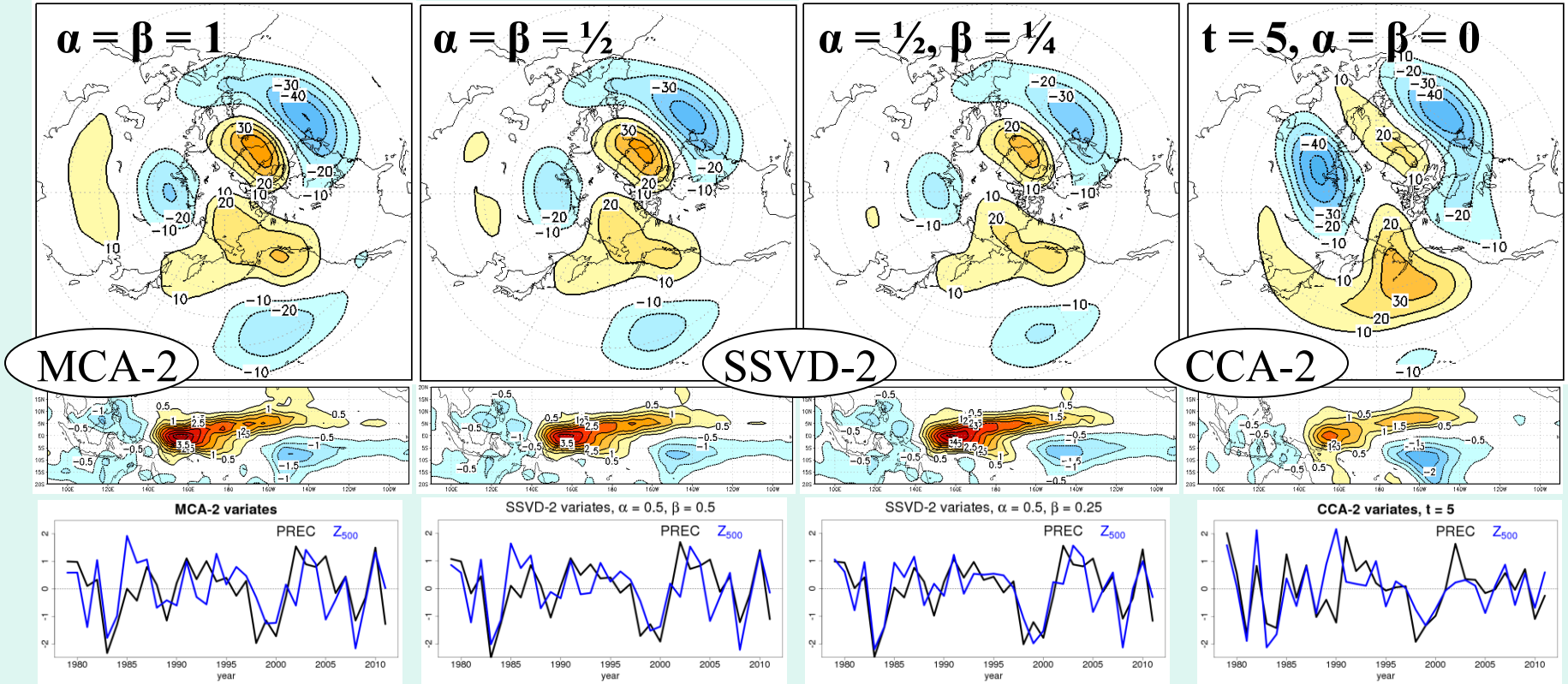
Signal-to-noise ratio *high* in tropics & *low* in extratropics → Use $\alpha > \beta$: scale down height more than precipitation

Linear ENSO signal



measure of relation	MCA-1	SSVD-1 α = β = ½	SSVD-1 α = ½, β = ¼	CCA-1 t = 5
SCF	0.48	0.49	**0.50**	0.49
COR	0.60	0.70	**0.75**	0.63

NH link with ENSO Modoki



measure of relation	MCA-2	SSVD-2 α = β = ½	SSVD-2 α = ½, β = ¼	CCA-2 t = 5
SCF	0.24	0.24	0.24	0.13
COR	0.51	0.65	**0.76**	0.59

Summary

- Scaling as prior constraint
  → Scaled SVD analysis, maximizes generic measure of relation $\left(\sigma_x{}^{\alpha}\rho_{x,y}\sigma_y{}^{\beta}\right)^2$
- Recovers traditional methods yet intermediate solutions tend to capture signal better
- Best choice of scaling parameters α, β proportional to true signal-to-noise ratio
- Stronger representation of ENSO & ENSO Modoki teleconnections (higher correlation & covariance)

Current & future work

- Alternative applications of scaling
  - prior constraint for optimizing other statistical measures
- Some promising variants of SSVD
  - rotations that maintain strict orthogonality
  - standardized scaling
- Climate applications: diagnostic studies, statistical downscaling
- Many potential applications exist across multiple disciplines

Thank You

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