A Diagnostic Study of the Time-Mean Atmosphere Over Northwestern Europe during Winter

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ABSTRACT

A diagnostic study has been performed to investigate the prospects for developing a time-averaged statistical-dynamical model for making long-range weather forecasts. Estimates are made of nearly all terms in the equations describing the evolution of the time-mean quantities $\bar{u}$, $\bar{v}$, $\bar{T}$, $\bar{\omega}$ and the horizontal second-order eddy statistics $\overline{u'^2}$, $\overline{v'^2}$, $\overline{u'v'}$, $\overline{u'T'}$ and $\overline{v'T'}$. These calculations were performed over northwestern Europe, using radiosonde observations of wind, temperature and height for the winter of 1976/1977. Geostrophic winds were estimated from objective analyses, while vertical velocities were determined with a quasi-geostrophic baroclinic model. For each equation, approximate balances are presented on the basis of these estimates.

In the equations for the mean quantities the time derivatives are more than one order of magnitude smaller than the unknown second-order eddy statistics. The same holds for the time derivatives of second-order eddy statistics compared with the unknown third-order and ageostrophic terms in the equations for these eddy fluxes. We therefore conclude that the system of time-averaged equations has no capability of describing the evolution of the atmosphere from one specific mean state to another mean state in the future—since for this purpose a closure of the system or a parameterization of second-order or third-order terms has to be extremely accurate. Even in the case in which only the stationary waves of the mean flow are treated, a higher order closure scheme does not seem to be feasible, for third-order terms and ageostrophic second-order terms are probably large and very difficult to parameterize. This implies that a preferable approach is to explore in greater depth the possibility of parameterizing the second-order statistics directly.

1. Introduction

In order to predict the changes in the large-scale atmospheric circulation pattern, we have to integrate the equations that describe the dynamical processes involved in these changes. The way in which we are allowed to modify the equations depends strongly on the integration time. If we integrate only one day ahead, we can look at the atmosphere as a quasi-geostrophic system. On a time-scale of several days, say, 3–14, we have to integrate deterministic equations that include diabatic heating as a very important driving mechanism. Finally, on the time-scales of climate fluctuations, dynamical processes have to be parameterized.

For the purpose of long-range weather forecasts we are interested in anomalies in the time-mean circulation (~1 month). But until now we hardly know which processes are responsible for such anomalies and to what extent.

Of course, we can attempt to make long-range weather forecasts with a general circulation model (GCM) in which all forcing functions are taken into account and all individual eddies and their influences on the mean flow are treated explicitly. Because the predictability of synoptic eddies is limited to about one week (Lorenz, 1969), we can at best expect to produce long-range forecasts of mean fields. Therefore the use of a GCM is very time-consuming.

An alternative way of making long-range weather forecasts is to treat the time-averaged flow explicitly, whereas transient systems, with periods less than the averaging period, are treated as a kind of turbulence. This turbulence then has to be related to the mean flow. Models of this type are called statistical dynamical (SD)[1] models.

The most direct way to derive equations for the time-mean flow is to take a time average of all terms in the primitive equations. The resulting equations contain a set of unknown variables. These "eddy-statistics" describe the effects of the transient eddies on the time-mean atmosphere.

At first sight, this way of attacking the problem seems much more elegant than a straightforward

[1] Here we adopt the terminology used by Kurihara (1970) and others, although the term "statistical dynamical model" is also used to describe the time evolution of the probability distribution of atmospheric states.

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integration with an explicit general circulation model. However, SD models will only be successful in making long-range weather forecasts if a reliable set of closure relations, diagnostic or prognostic, can be derived. It is well known that the transient eddies contribute substantially to the maintenance of the time-mean fields (Lorenz, 1967).

Most of the models discussed in the literature are zonally averaged. One of the few time-averaged and nonzonally averaged models was constructed by Adem (1964, 1970). Adem uses a time-dependent heat-balance model in which only the thermodynamic equation is used. The unknown eddy heat flux is assumed to be of a diffusive form. Recently, Vernekar and Chang (1978) presented a linear time-averaged steady-state SD model which describes stationary perturbations. Their equations deal with both dynamical and thermodynamical processes and, again, eddy fluxes are approximated by diffusion terms.

The use of a diffusion concept in order to parameterize second-order eddy statistics is not based on much observational evidence. From diagnostic studies it becomes clear that it is very difficult to find closure relations. In order to avoid problems with second-order eddy statistics, one can formulate equations for their evolution in time. In doing so we introduce new unknowns: third-order eddy statistics—this in the hope that these terms will be easier to deal with. Until now very little work has been published on this possibility. Exceptions are papers by Saltzmann *et al.* (1961), van den Dool (1975) and Savijärvi (1976).

Apart from very difficult problems concerning the closure of the system of equations, one must deal with many other questions: Is there an optimum averaging period? What are leading terms in the equations and can any of the terms be neglected?

There are many papers that touch on these problems. The purpose of some of these studies is to obtain a better insight in the *maintenance* of certain quantities (see, e.g., Lau *et al.*, 1978). But because *prediction* of time-mean quantities is the ultimate goal, one should also investigate the possibilities of prognostic equations. Therefore it makes sense to study a rather short period of a few months, instead of many years or many seasons.

A complete observational survey of the terms in all equations might be useful in the description of the time-averaged atmosphere. In this paper calculations are presented of the orders of magnitude of nearly all terms in the equations describing time mean quantities $\bar{u}$, $\bar{v}$, $\bar{T}$, $\bar{\omega}$ and the equations describing the horizontal second-order eddy statistics $\overline{u'^2}$, $\overline{v'^2}$, $\overline{u'v'}$, $\overline{u'T'}$ and $\overline{v'T'}$. These calculations have been performed over northwestern Europe which, of course, is a rather small domain. We used radiosonde observations of wind, temperature and height at 10 standard levels for one winter.

It is hoped that scaling considerations will provide a better insight into the following problems. First, the closure problem: can we neglect or easily parameterize the third-order terms in the equations for second-order terms? If this were the case, then it is advantageous to close the system at third order, despite the fact that we introduce another five nonlinear differential equations at the same time. And second, how can we find a way to modify the original set of equations such that they become easier to use?

2. Formulation of the problem for the time-mean atmosphere

The set of equations for dry air commonly used in meteorology consists of the horizontal momentum balance, the hydrostatic assumption, the continuity equation, the first law of thermodynamics, and the equation of state. Together these equations form a closed set in six unknowns, u, v, ω, T, Φ and α; the horizontal and vertical wind, temperature, geopotential height and specific volume, respectively. Friction and diabatic heating have to be prescribed.

In order to obtain equations for the time-averaged flow we have first to define a time average (see Appendix for list of symbols).

$$\bar{F}(t) = \frac{1}{\Delta t}\int_{t-1/2\Delta t}^{t+1/2\Delta t} F(t')dt'. \tag{2.1}$$

At any time F can be separated into its time average and a deviation, thus,

$$F = \bar{F} + F'. \tag{2.2}$$

If (2.2) is substituted for all variables in all basic equations and, consequently, the time-averaging procedure is applied, then we obtain equations for the evolution of $\bar{u}$, $\bar{v}$, $\bar{\omega}$, $\bar{\Phi}$, $\bar{T}$ and $\bar{\alpha}$. In addition, we assume that $\overline{F'} = 0$, which is not necessarily true for running means.

The equations for averaged horizontal motion are completely comparable to the usual equations of motion, except that large-scale Reynolds' stresses are included now as additional forces. In the same way we find in the thermodynamic equation the large-scale eddies as additional heating. The subsequent terms of these three equations can be found in Tables 1 and 2.

We now have a system of six equations in 14 unknown variables. These variables are $\bar{u}$, $\bar{v}$, $\bar{\omega}$, $\bar{T}$, $\bar{\Phi}$, $\bar{\alpha}$ and the eddy statistics $\overline{u'^2}$, $\overline{v'^2}$, $\overline{u'v'}$, $\overline{u'T'}$, $\overline{v'T'}$, $\overline{u'\omega'}$, $\overline{v'\omega'}$ and $\overline{\omega'T'}$. Obviously, we do not have a closed set of equations anymore. Because, in general, the eddy statistics are found to

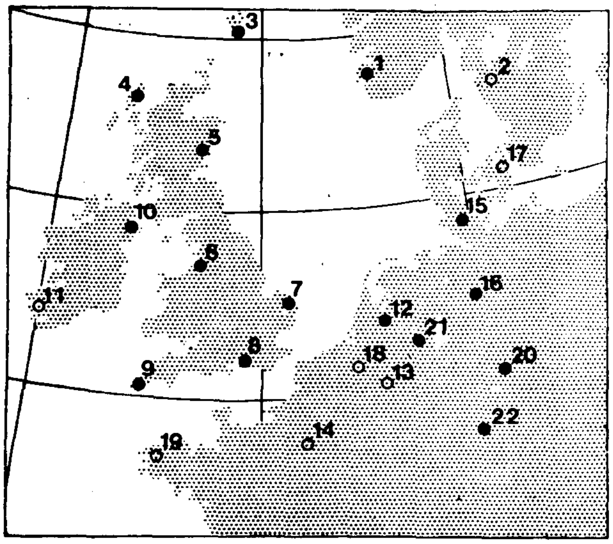

FIG. 1. The northwestern European area where the computations were performed at radiosonde stations 1–22. Stations that hardly could be used are denoted by open circles.

be important, one cannot simply neglect them. Unfortunately, satisfactory parameterizations of the second-order eddy statistics are not available. Therefore, we shall try an alternative approach.

It is possible to add new equations for the time derivative of the horizontal eddy fluxes and variances. Such equations can be derived by simple manipulations with the non-averaged equations. For example, we multiply the u-momentum equation by u' and take a time average afterward, then we obtain a prognostic equation for the variance of u. The subsequent terms of the equations for $\overline{u'^2}$, $\overline{v'^2}$, $\overline{u'v'}$, $\overline{u'T'}$ and $\overline{v'T'}$ can be found in Tables 3 to 7.

The structure of these five equations is roughly the same and consists of five groups of terms. The first group represents advection of the eddy flux quantities by the mean wind. In the second group we find creation and destruction terms (ageostrophic terms, friction, diabatic heating). In the third group the influence of the curvature of the coördinate system is described, while the fourth group represents the interaction with the mean flow. Finally, the fifth group consists of the third-order terms, which describe the mean effect of advection of eddy flux quantities by eddy motion.

A derivation of similar equations for vertical fluxes like $\overline{u'\omega'}$ is impossible. The reason for this is that we have no prognostic equation for ω because of hydrostatic equilibrium. Through the introduction of the five eddy flux equations, we have to accept 17 new unknown variables. Eleven of these are third-order terms: $\overline{u'^3}$, $\overline{v'^3}$, $\overline{u'^2v'}$, $\overline{u'v'^2}$, $\overline{u'^2\omega'}$, $\overline{v'^2\omega'}$, $\overline{u'v'\omega'}$, $\overline{u'^2T'}$, $\overline{v'^2T'}$, $\overline{u'v'T'}$, $\overline{u'\omega'T}$ and $\overline{v'\omega'T'}$. The remaining unknowns have been expressed as ageostrophic terms: $\overline{u'v'_{ag}}$, $\overline{v'u'_{ag}}$, $\overline{u'u'_{ag}}$, $\overline{v'v'_{ag}}$, $\overline{T'v'_{ag}}$ and $\overline{T'u'_{ag}}$. It is more useful to write them as ageostrophic expressions than to maintain pressure and Coriolis terms separately. Because of the approximate geostrophic balance the latter two are, in general, very large but only their small imbalance is relevant in the equations.

If we consider all terms involving friction and diabatic heating as known functions, then the problem as a whole is formulated as a set of 11 equations in 31 unknowns. At first sight the result is a very complex description even though we do not have to prescribe second-order eddy heating explicitly. To find closures we shall have to parameterize third-order terms, second-order vertical fluxes and ageostrophic terms. There is no doubt that this is a tremendous task.

There is very little information concerning the importance of third-order terms in the equations mentioned above. In a study of meridional transport of eddy kinetic energy Saltzmann (1961) found the third-order contribution to be important. Van den Dool (1975) found that the third-order terms in the eddy kinetic energy equation, though smaller than the leading terms, cannot be neglected. Moreover, a diffusion parameterization is not useful. Savijärvi (1976) arrived at the same conclusions. On the other hand, Lau *et al.* (1978) state that the third-order terms are small in the balance equation for the eddy momentum flux $\overline{u'v'}$.

Even less direct observational information is available concerning large-scale vertical eddy fluxes of momentum and heat. Holopainen (1964, 1973) studied the balance of time-averaged horizontal momentum over England. He calculated mean vertical fluxes with the kinematic and adiabatic methods. Holopainen calculated that the vertical fluxes are about one order of magnitude smaller than the leading terms in the equations. For zonally averaged flow vertical eddy fluxes have been determined as a residue; Hantel (1976) and Hantel and Hacker (1978) found that in this case vertical fluxes by eddies of all scales may become important.

Finally, the ageostrophic terms are undoubtedly important. For example, all kinetic energy in the atmosphere is produced by ageostrophic winds; in the context of the horizontal momentum flux equation Lau *et al.* (1978) find ageostrophic terms to be of major importance.

3. Data and analysis

For our computations we chose 22 radiosonde stations in Northwestern Europe (see Fig. 1). Twice daily measurements of wind, temperature and geopotential height at 10 levels during the period 0000 GMT 17 December 1976 to 1200 GMT 16 March 1977 were used. The levels are the surface and

850, 700, 500, 400, 300, 250, 200, 150 and 100 mb. For our purpose it is convenient to look at these raw data as a collection of time series. So, at 22 stations and 10 levels we have time series of u, v, T and z, each consisting of at most 180 observations. Each time series was first checked, and missing data were filled in by linear interpolation. Elements that deviated more than four standard deviations from the mean were considered missing. If four or more consecutive data were missing, the complete series was rejected. Also, if the missing data totaled 10%, such a series was not used anymore.

For these reasons many series could not be used and for several stations it was hard to find any reliable series. It turns out that starting with 22 stations yields only 15 stations with more or less complete upper air data (see Fig. 1). In all maps to be presented in the next section, those quantities that could be computed after the check are plotted near the stations.

As far as possible for each station, the following mean terms and eddy statistics were computed:

$\bar{u}$, $\bar{v}$, $\bar{T}$, $\bar{z}$,

$\overline{u'^2}$, $\overline{v'^2}$, $\overline{u'v'}$, $\overline{u'T'}$, $\overline{v'T'}$,

$\overline{u'^3}$, $\overline{v'^3}$, $\overline{u'^2v'}$, $\overline{u'v'^2}$, $\overline{u'^2T'}$, $\overline{v'^2T'}$, $\overline{u'v'T'}$

$$\frac{\partial \bar{u}}{\partial t}, \frac{\partial \bar{v}}{\partial t}, \frac{\partial \bar{T}}{\partial t}, \frac{\partial \overline{u'^2}}{\partial t}, \frac{\partial \overline{v'^2}}{\partial t},$$

$$\frac{\partial \overline{u'v'}}{\partial t}, \frac{\partial \overline{u'T'}}{\partial t}, \frac{\partial \overline{v'T'}}{\partial t}.$$

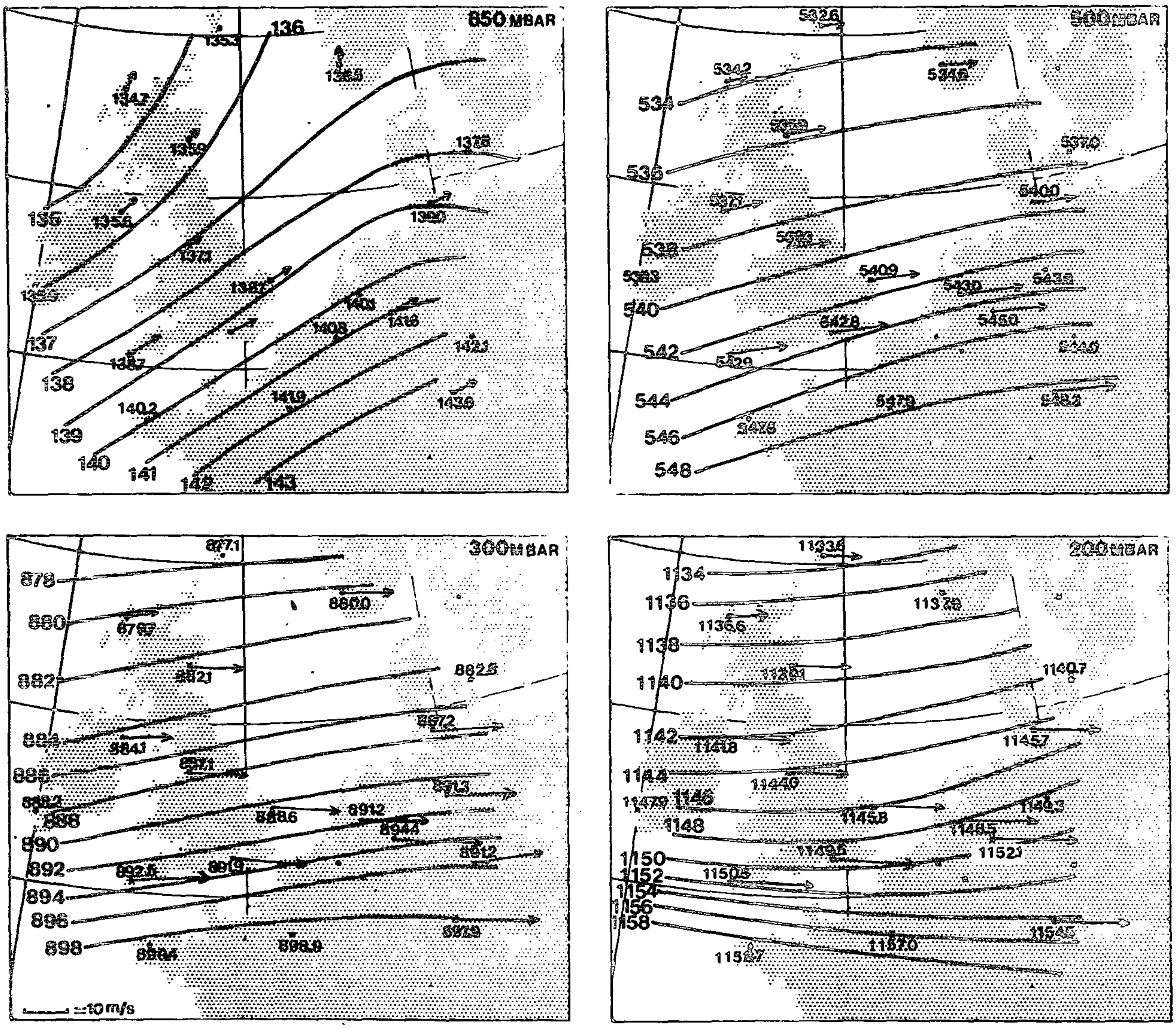

FIG. 2. Spatial distribution of time-mean geopotential height ($\bar{z}$) during the winter of 1976/77 at four pressure levels. Contour interval is 2 dam, except for 850 mb where it is 1 dam. Arrows indicate time-averaged wind vectors; the scale is indicated in the lower left corner.

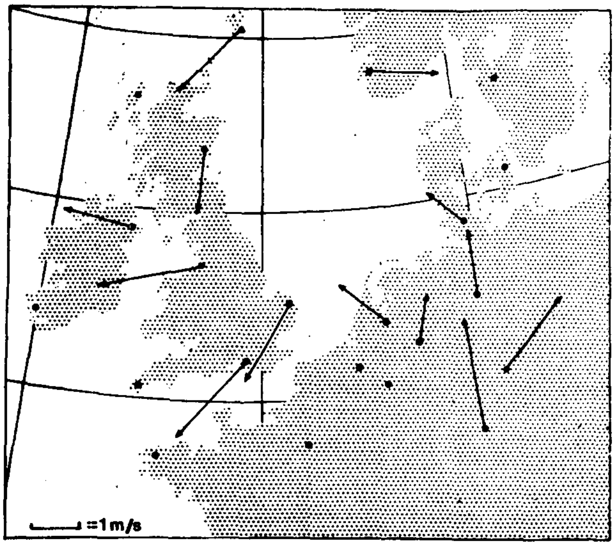

FIG. 3. Ageostrophic winds at 300 mb, determined with objective height analyses. Note that scale is 1 m s^{-1}, compared with 10 m s^{-1} in Fig. 2.

All terms were computed with an averaging period of 5, 10, 15, 30, 45 and 90 days. We hoped that, for example, the third-order terms would be very small in some properly chosen averaging period. However, from our computations we could not find the existence of such a period. Hence we shall restrict ourselves to a description of the results for an averaging period of 90 days.

From hand-analyses the gradients of mean and eddy quantities were estimated at four levels, *viz.*, 200, 300, 500 and 850 mb.

In order to have an estimate of large-scale vertical mean and eddy fluxes, vertical velocities were derived from a filtered quasi-geostrophic baroclinic three-level model. This model is in operational use in the Netherlands Weather Service; it provides vertical velocities (ω) at three pressure levels: 300, 500 and 850 mb. From objective analyses of geopotential height at 0000 and 1200 GMT time series of vertical velocities were composed for the locations of the 15 radiosonde stations. If we combine the direct observations of u, v, T and the pseudo-observations of ω, the following eddy and mean quantities can be computed:

$\bar{\omega}$,

$\overline{\omega'u'}$, $\overline{\omega'v'}$, $\overline{\omega'T'}$ and

$\overline{u'^2\omega'}$, $\overline{v'^2\omega'}$, $\overline{u'v'\omega'}$, $\overline{u'\omega'T'}$, $\overline{v'\omega'T'}$.

The same objective analysis of geopotential height has been used to compute geostrophic winds at the locations of the 15 stations at the 300, 500 and 850 mb levels. Combined with actually observed winds this allows us to estimate $\overline{u'v'_{ag}}$, $\overline{v'u'_{ag}}$, $\overline{u'u'_{ag}}$, $\overline{v'v'_{ag}}$, $\overline{T'v'_{ag}}$ and $\overline{T'u'_{ag}}$.

4. Reliability of the eddy statistics

We investigated the sensitivity of the horizontal eddy statistics for errors in the wind components. For this purpose, random errors of 2 m s^{-1} were added to both the u and v component. The resulting new statistics showed small deviations of a few per cent.

It is likely, however, that measuring errors are dependent on wind strength. We have simulated this dependence by adding to both velocity components random errors to a maximum of 10% of the respective components. In this case the resulting differences amounted to 20% for $\overline{u'^2}$, $\overline{v'^2}$, $\overline{u'T'}$ and $\overline{v'T'}$, 50% for $\overline{u'v'}$, and 100% for the third-order statistics.

It is well known that the determination of vertical transports, using ω-time series derived with direct or indirect methods, results in large errors. There is no doubt that this applies also to our ω series. The first reason is that only synoptic-scale vertical motion is considered. One cannot *a priori* neglect vertical transports by subsynoptic motion (Palmén and Newton, 1969). The second reason is the coarse resolution of our model, both horizontally and vertically. Moreover, surface friction, diabatic heating and ageostrophic effects are not taken into account. Therefore the vertical fluxes, even those averaged over a whole winter, contain large errors. Nevertheless, as far as the 90-day mean vertical velocity is concerned, it was found that there is a good agreement with vertical velocities derived from the 90-day mean divergence of the observed winds.

The last group of terms, which may contain large errors, are the ageostrophic terms. The ageostrophic wind is a small difference between two large quantities, which are poorly known, at least for this purpose. We can illustrate this for the time-mean geostrophic wind, because we have two estimates. The first, of course, is the average of the geostrophic wind derived from the objective analyses and used in all computations. A second estimate can be derived from a subjective analysis of time-averaged geopotential height ($\bar{z}$). As will be visualized in the Section 5 (see Figs. 2 and 3) these two estimates show differences which are important in the calculation of the ageostrophic wind.

From the foregoing information it is clear that the composition of a reliable set of statistics is a tremendous task. This seems to be especially true for the third-order terms, vertical transport terms and ageostrophic terms. Hence, one should be very careful in the interpretation of these statistics.

5. Mean state of the atmosphere over northwestern Europe during the winter of 1976/1977

We start our discussion of the results with a description of the time-mean atmosphere. In Fig. 2 the distribution of time-mean geopotential height is given for the 850, 500, 300 and 200 mb levels. Time-mean wind vectors are plotted as arrows. From these four maps it becomes clear that the time-averaged atmosphere is rather baroclinic; the mean winds veer with height, especially in the lower half of the troposphere.

At 200 and 300 mb there is considerable cross-isobaric flow; at many stations there is a small wind component toward high pressure. It turns out that the determination of the ageostrophic wind component is very sensitive to the choice of the height-field analysis. In Fig. 2 we used a subjective analysis derived from mean heights only; Fig. 3 shows ageostrophic winds determined with the use of our objective height analyses. In these latter analyses wind measurements are used to estimate local gradients of the height field. However, one must doubt whether this is a real improvement for our purpose; it is far less clear from Fig. 3 that there is a general cross-isobaric flow toward high pressure. This is an illustration of the uncertainties in the computation of ageostrophic winds mentioned earlier.

Fig. 4 gives the spatial distribution of time-mean temperature at four levels. Again, the arrows are mean wind vectors. The most significant features of

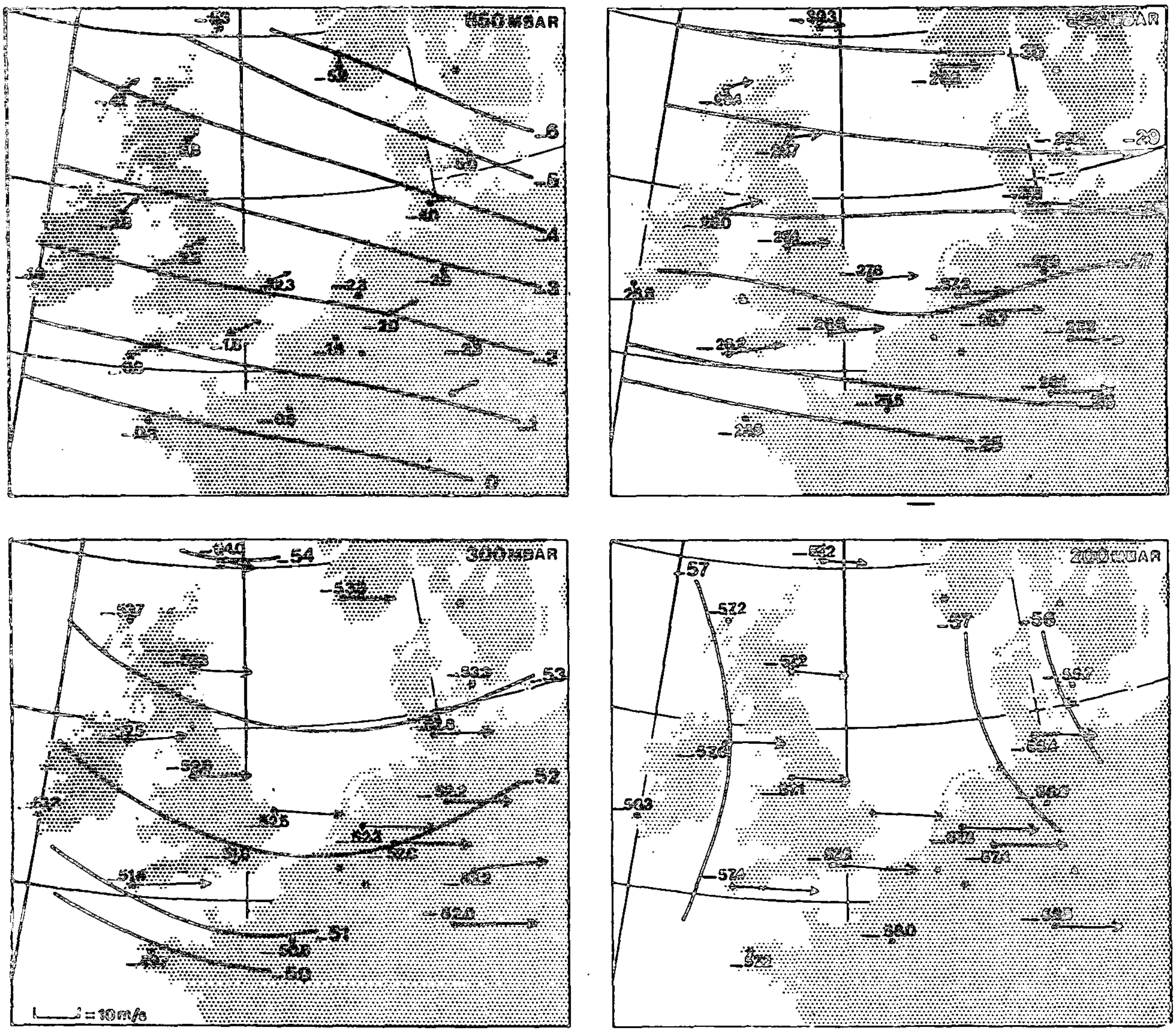

FIG. 4. Spatial distribution of time-mean temperature ($\bar{T}$) during the winter of 1976/77 at four pressure levels. Isotherms are drawn at intervals of 1 K. As in Fig. 2, arrows represent time-mean wind vectors.

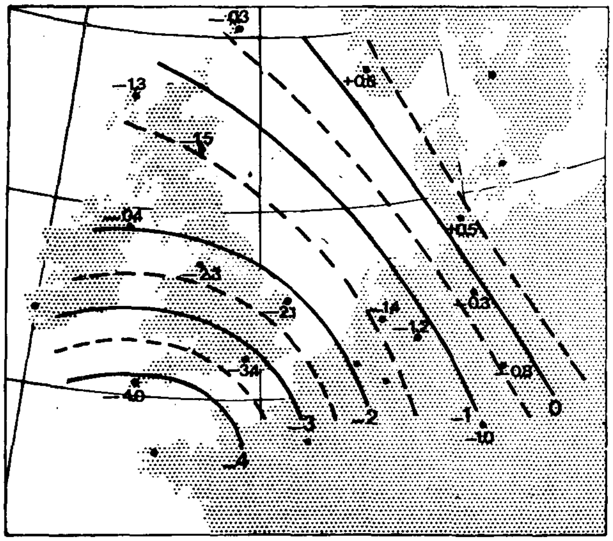

FIG. 5. Time-mean vertical velocity distribution at 500 mb ($\bar{\omega}$) during the winter of 1976/77. Negative values indicate upward large-scale motion. Isolines are drawn at intervals of 10^{-2} N m^{-2} s^{-1}.

these maps are the strong warm air advection at 850 mb and the gradual decrease of horizontal temperature gradients with height. As usual, at 200 mb the gradients become very weak or reverse sign.

The veering of the mean wind in the lower layers and the apparent phase difference between contours and isotherms indicate that standing eddies have a baroclinic structure.

The distribution of the vertical velocity $\bar{\omega}$ at 500 mb is displayed in Fig. 5. Most of the area is characterized by upward motion. The heating due to warm advection is counteracted by cooling due to rising motion. Over the southern part of the British Isles these effects are equivalent to a temperature rise of 2.0 K day^{-1} and a temperature decrease of about 1.2 K day^{-1}, respectively.

As an example, the vertical mean profiles of $\bar{u}$, $\bar{v}$ and $\bar{T}$ of station 9 (Camborne, Southwest England) are shown in Fig. 6. All profiles are compared with the averages at the 50° latitude circle (Oort and Rasmusson, 1971). The winds can also be compared with geostrophic approximations at three levels. The zonal wind profile has a very distinct maximum at 250 mb, which is not uncommon for this latitude. Although much stronger than the zonal average, zonal winds in the upper troposphere are subgeostrophic by a few meters per second. This was also found by Holopainen for the autumn of 1954 (Holopainen, 1964). In the lowest 150 mb the shear in $\bar{u}$ is remarkably stronger than in the zonal average.

Except at 850 mb the meridional wind is close to its geostrophic value. This of course, is in contrast with the zonally averaged meridional wind, which is entirely ageostrophic. The meridional wind found over Southwest England is about one order of magnitude larger than the zonally averaged value. Virtually all air moves northward in this area. Obviously, the almost geostrophic meridional mass circulation in horizontal standing eddies dominates the overturning in the ageostrophic Ferrel cell.

The mean temperature profile in the lower half of the atmosphere is locally much steeper than the corresponding zonal average. The temperatures over 500 mb are almost the same but the surface temperature is about 10 K above the zonal average.

An important aspect is the observed vertical shear of the zonal wind. Is the time-mean flow baroclinically stable in the traditional sense? This is an important question in the context of SD modeling. The baroclinic stability properties were investigated with the help of a linearized version of the three-level quasi-geostrophic baroclinic model. Using static stabilities and wind shear observed over southwest England, wavelengths ranging from 1750 to 5000 km turned out to be unstable. Minimum doubling time of these waves amounts to 32 h. But we are sure that the mean atmosphere does not show any wave growth in this part of the spectrum. How does one deal with this in an SD model? Two arguments can be put forward. The first, of course, is the transient character of such unstable waves. Transient waves do not appear in time-mean flow and therefore a time-averaged model should be

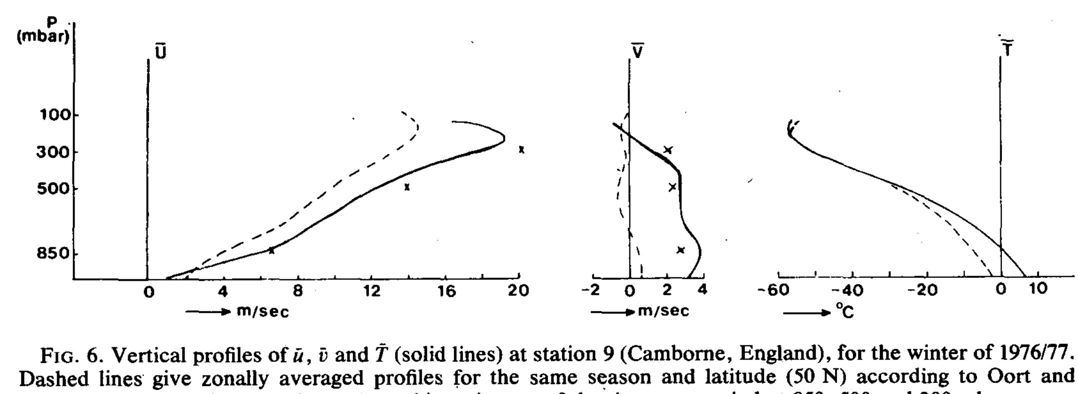

FIG. 6. Vertical profiles of $\bar{u}$, $\bar{v}$ and $\bar{T}$ (solid lines) at station 9 (Camborne, England), for the winter of 1976/77. Dashed lines give zonally averaged profiles for the same season and latitude (50 N) according to Oort and Rasmusson (1971). Crosses give geostrophic estimates of the time-mean wind at 850, 500 and 300 mb.

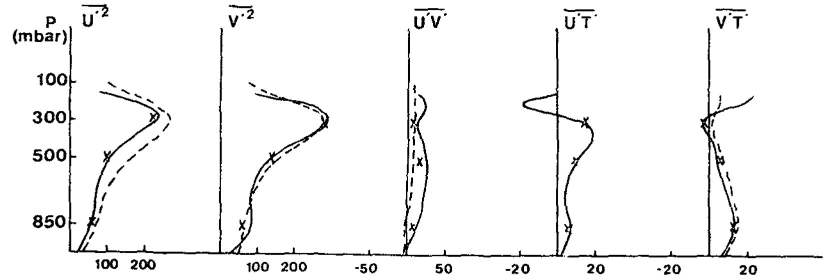

FIG. 7. Vertical profiles of $\overline{u'^2}$, $\overline{v'^2}$, $\overline{u'v'}$, $\overline{u'T'}$ and $\overline{v'T'}$ at station 9 (Camborne, England) for the winter of 1976/77. Dashed lines give zonally averaged values for 50°N according to Oort and Rasmusson (1971). Crosses give geostrophic fluxes at three levels. Units are $m^2\ s^{-2}$ and $m\ K\ s^{-1}$.

constructed in such a way that these waves are suppressed or made nonexistent. A simple way to do this is the use of smoothing operators or, alternatively, heavy truncation of the spectrum. A second possibility is that the eddy terms in the model-equations drastically change the stability properties in such a way that wave growth does not occur anymore at those wavelengths.

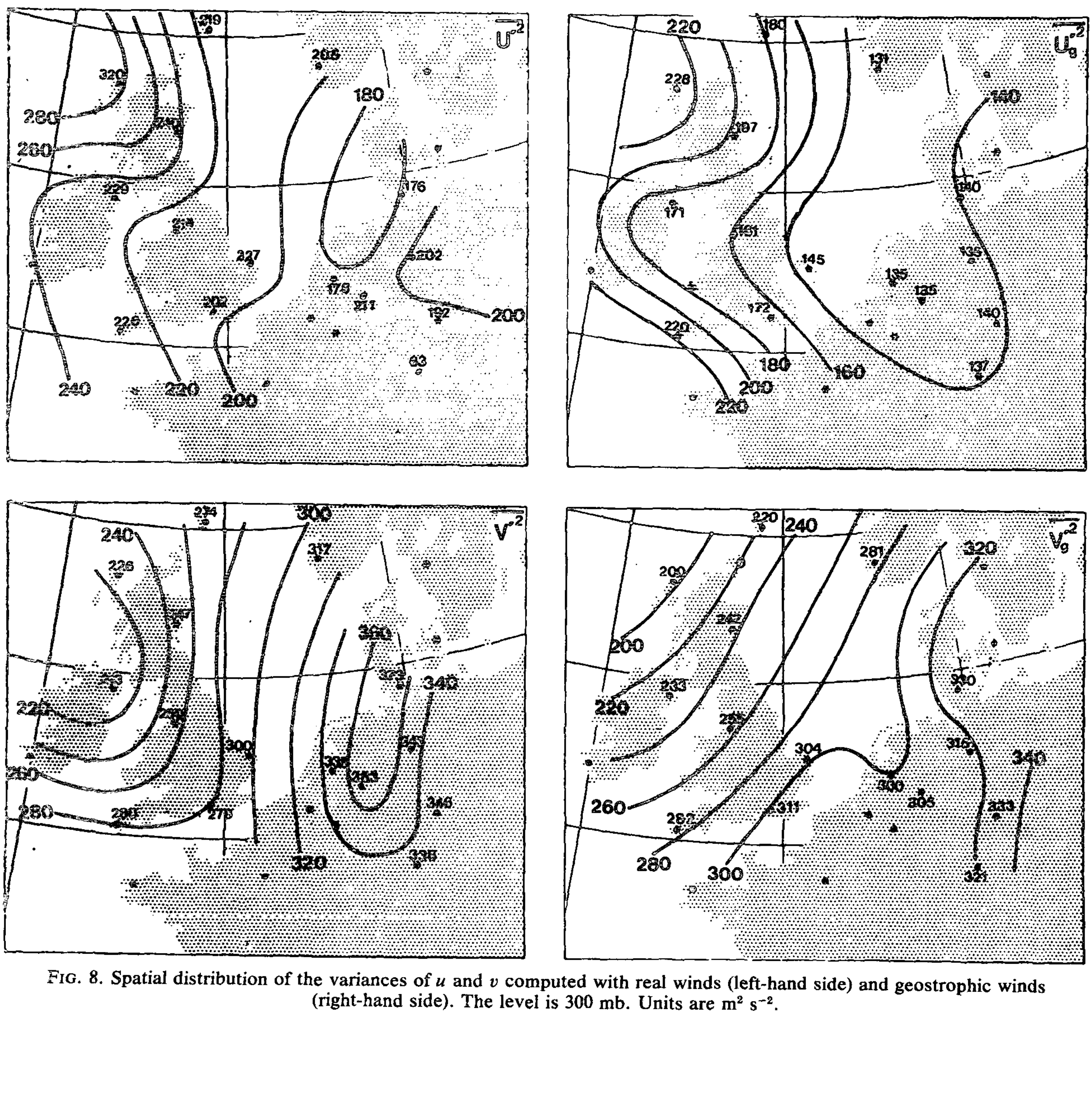

FIG. 8. Spatial distribution of the variances of u and v computed with real winds (left-hand side) and geostrophic winds (right-hand side). The level is 300 mb. Units are $m^2\ s^{-2}$.

6. Second-order eddy statistics over northwestern Europe

The influence of transient weather systems on the evolution of the mean atmosphere is represented as horizontal and vertical gradients of eddy fluxes of heat and momentum. An example of the vertical profiles of horizontal eddy terms is shown in Fig. 7. As in the previous section, data of station 03808 (Camborne, England) were used for this purpose. As far as possible a comparison is made with zonally averaged profiles at 50°N (Oort and Rasmusson, 1971) and with geostrophic eddy terms at three levels. Data of Oort and Rasmusson suggest that the profiles of $\overline{u'^2}$ and $\overline{v'^2}$ are very nearly the same. This means that large-scale turbulence is isotropic, which is in agreement with the almost negligible $\overline{u'v'}$ at these latitudes. However, our results indicate that local deviations of isotropy occur; at jet-stream level the variances of u and v may differ by 25%. Furthermore, the meridional flux of zonal momentum is substantially larger than the zonal average. These features can also be judged from maps of $\overline{u'^2}$, $\overline{v'^2}$ and $\overline{u'v'}$ at 300 mb (Figs. 8 and 9).

The local (southwest England) meridional eddy temperature flux profile shows a maximum at 850 mb, a minimum at 300 mb and again strong poleward fluxes at stratospheric levels. This is qualitatively in good agreement with data of Oort and Rasmusson except that the stratospheric flux in this study is much stronger. The zonal eddy heat flux is predominantly eastward in the troposphere; this is

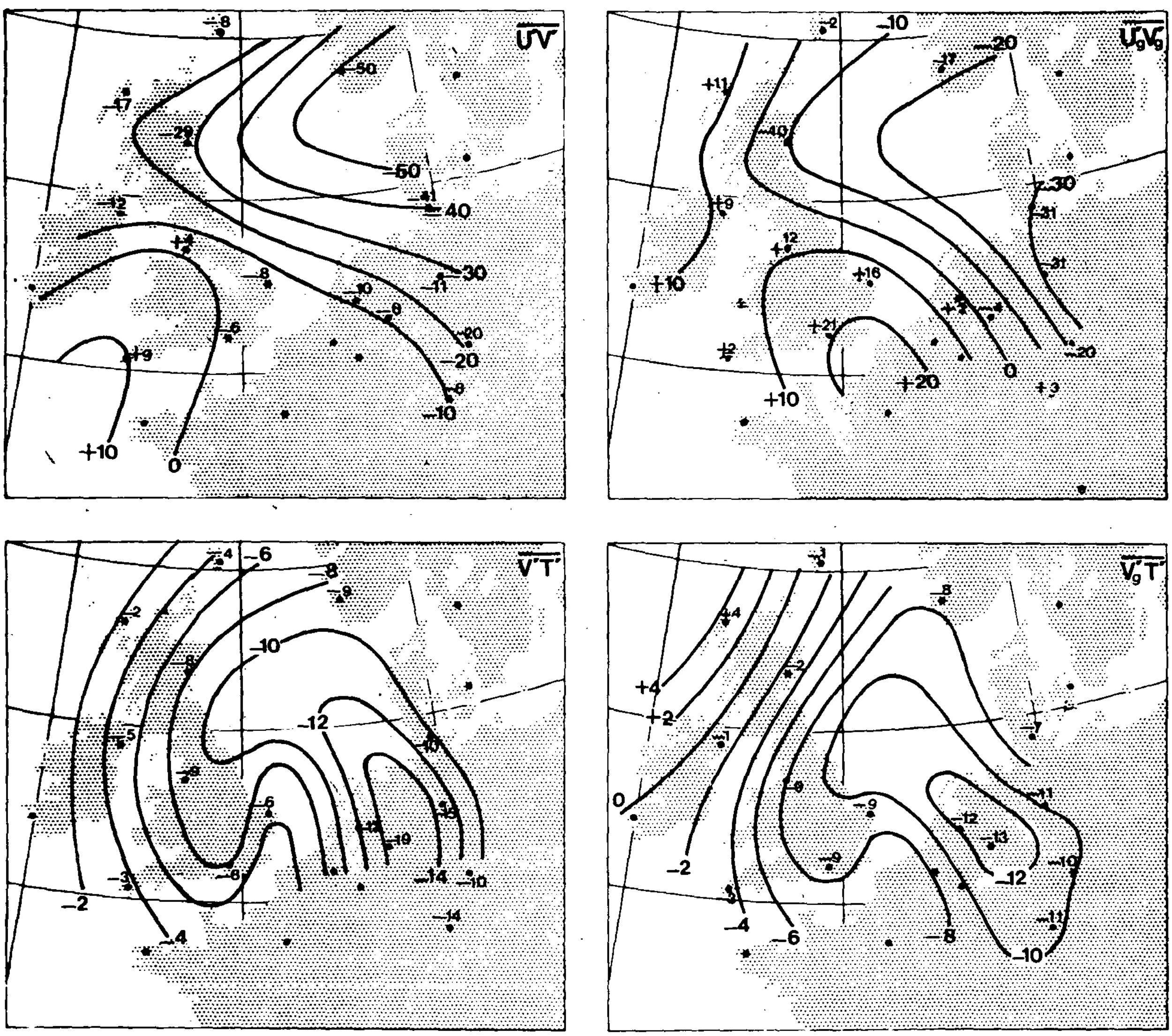

FIG. 9. Spatial distribution of meridional momentum and heat fluxes computed with real winds (left-hand side) and geostrophic winds (right-hand side). The level is 300 mb. Units are $m^2 s^{-2}$ and $m K^{-1} s^{-1}$, respectively.

undoubtedly related to the enormous heat input of the ocean into the atmosphere, which is consequently swept into the continent where the radiation balance is negative in winter.

A peculiar phenomenon is the negative value of $\overline{v'T'}$ at 300 mb in Fig. 7. It is probably a real feature, as it is consistently shown at all radiosonde stations (Fig. 9). The northern latitudes on the average being colder than the southern, this would mean a countergradient meridional eddy heat flux at 300 mb (the zonal eddy heat flux $\overline{u'T'}$ is nearly parallel to the isotherms). This result is confirmed by Lau (1978) and also by unpublished maps of Oort and Rosenstein. In fact, countergradient heat fluxes in the higher troposphere over the western parts of the continents in winter is a normal though unexplained feature of the general circulation. It is remarkable that if we cross the tropopause the rapid change in $\overline{v'T'}$ is accompanied by a similar change in $\overline{u'T'}$.

If we compare eddy fluxes computed with real winds to those computed with geostrophic winds, there is a general resemblance. This is certainly the case in Fig. 7. However, larger differences occur at other places and, moreover, small differences may lead to meaningful differences in the gradients of these fluxes. In Figs. 8 and 9 horizontal distributions of $\overline{u'^2}$, $\overline{v'^2}$, $\overline{u'v'}$ and $\overline{v'T'}$, and $\overline{u_g'^2}$, $\overline{v_g'^2}$, $\overline{u_g'v_g'}$ and $\overline{v_g'T'}$ at 300 mb are given. Due to the noisy character of fluxes based on raw data, it is difficult to make a spatial analysis. The subjective analysis includes a certain degree of smoothing; sometimes we had to reject one or two stations. If we compare the analyses in Figs. 8 and 9, it can be seen that the gross features are the same for real and geostrophic winds. The computation of horizontal gradients may lead to quite different results, however. It is not clear which of the two estimates of gradients is to be preferred. Fluxes based on real observations contain more small-scale structures, but some of this may be noise. Geostrophic fluxes have better spatial coherence and fewer small-scale features.

Fig. 10 shows the spatial distribution of $\overline{u'\omega'}$ and $\overline{v'\omega'}$ at 500 mb. Both zonal and meridional momentum are transported mainly upward by synoptic eddies, the latter transport being almost three times larger than the first. The order of magnitude of the vertical eddy flux of zonal momentum at 500 mb (10.10^{-2} N m^{-1} s^{-2}) compares well with results of Holopainen (1964).

7. Estimates of the terms in the equations for mean momentum and temperature

The statistical properties of time-averaged momentum and temperature balance equations have been investigated in several general circulation studies. For the zonally averaged conditions a rather complete picture can be derived from the atmospheric statistics of Oort and Rasmusson (1971). However, knowledge of the local features of the general circulation is far less complete. For a non-zonally averaged atmosphere a quantitative evaluation of the mean momentum and temperature equation has been given by Holopainen (1964, 1973) and Savijärvi (1966, 1977). Savijärvi's study is comparable to ours in the sense that his formulation of the equations is the same. However, he could not compute the vertical flux convergence terms, except terms involving $\bar{\omega}$ in the thermodynamic equation.

Our estimates of the various terms in both momentum equations are given in Table 1. As an estimate of all terms we have taken in a subjective way

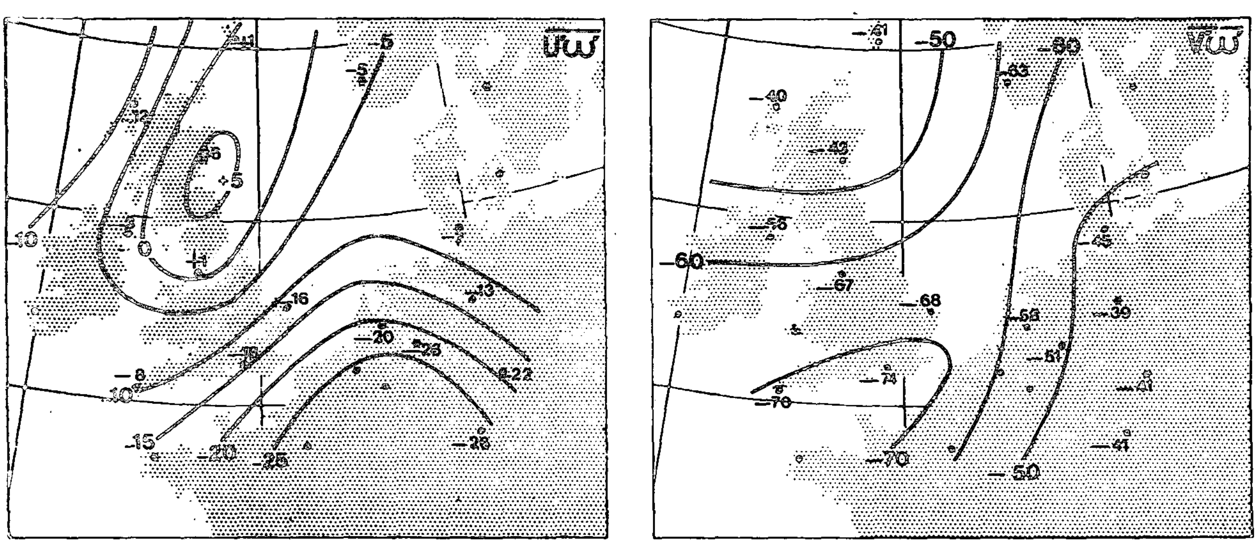

FIG. 10. Spatial distribution of vertical eddy fluxes of zonal (left-hand side) and meridional (right-hand side) momentum. The level is 500 mb. Units are 10^{-2} N m^{-1} s^{-2}.

TABLE 1. Estimates of the terms in the equations for mean horizontal motion at four pressure levels. Units are 10^{-5} m s^{-2}. The complete equations can be read from the top to the bottom of the table.

		850	500	300	200			850	500	300	200
	$\frac{\partial \bar{u}}{\partial t}$	0.3	0.5	0.6	0.5		$\frac{\partial \bar{v}}{\partial t}$	0.3	0.5	0.6	0.5
+	$\bar{\mathbf{V}}\cdot\nabla\bar{u}$	4	7	9	9	+	$\bar{\mathbf{V}}\cdot\nabla\bar{v}$	3	5	18	7
+	$\bar{\omega}\frac{\partial \bar{u}}{\partial p}$		0.8			+	$\bar{\omega}\frac{\partial \bar{v}}{\partial p}$		0.4		
−	$f\bar{v}_{ag}$	12	8	18		+	$f\bar{u}_{ag}$	10	14	24	
−	$\bar{F}_x$					−	$\bar{F}_y$				
−	$\bar{u}\,\bar{v}\frac{\tan\varphi}{a}$	0.3	0.4	0.6	0.6	+	$\bar{u}^2\frac{\tan\varphi}{a}$	1	2	5	5
−	$2\overline{u'v'}\frac{\tan\varphi}{a}$	0.4	0.8	2	0.8	+	$(\overline{u'^2}-\overline{v'^2})\frac{\tan\varphi}{a}$	1	2	3	2
+	$\frac{\partial \overline{u'^2}}{\partial x}$	10	8	15	8	+	$\frac{\partial}{\partial x}\overline{u'v'}$	2	5	5	10
+	$\frac{\partial \overline{u'v'}}{\partial y}$	2	6	10	4	+	$\frac{\partial}{\partial y}\overline{v'^2}$	10	10	12	8
+	$\frac{\partial \overline{u'\omega'}}{\partial p}$		0.5			+	$\frac{\partial}{\partial p}\overline{v'\omega'}$		2		
=	0					=	0				

absolute values which are considered to be representative for the area. In most cases the estimates are close to the extreme values except when the extreme was suspect. This procedure has been followed also in Tables 2–7, discussed in the next sections. It is clear that no balance of all terms can be made from which friction or diabatic heating follows as a residue. The convergence of vertical fluxes is available only at 500 mb, while the ageostrophic terms are given only at 850, 500 and 300 mb.

TABLE 2. As in Table 1 except for mean temperature. Units are 10^{-5} K s^{-1}.

		850	500	300	200
	$\frac{\partial \bar{T}}{\partial t}$	0.1	0.1	0.2	0.2
+	$\bar{\mathbf{V}}\cdot\nabla\bar{T}$	3	3	5	4
−	$\bar{\omega}\left(\frac{R\bar{T}}{pc_p}-\frac{\partial \bar{T}}{\partial p}\right)$		2		
−	$\frac{\bar{Q}}{c_p}$				
−	$\overline{v'T'}\frac{\tan\varphi}{a}$	0.3	0.2	0.3	0.7
+	$\frac{\partial}{\partial x}\overline{u'T'}$	1	1	2	2
+	$\frac{\partial}{\partial y}\overline{v'T'}$	1	2	2	1
+	$\frac{\partial}{\partial p}\overline{\omega'T'}$		0.2		
−	$\frac{R}{pc_p}\overline{\omega'T'}$		0.05		
=	0				

It can easily be seen from Table 1 that the time rate of change of mean momentum is very small. We therefore conclude that the equations are essentially maintenance equations. The dominating balance in the equations is the geostrophic balance, which is eliminated by taking the ageostrophic term. This ageostrophic residue balances approximately with the advection part of the acceleration term, the eddy forces and possibly friction.

Both vertical mean and eddy fluxes are about one order of magnitude smaller than the leading terms in this approximate balance. In the zonal momentum equation the curvature terms are very small; however, they are not negligible in the y direction.

Estimates of the terms in the equation for mean temperature are given in Table 2.

From the information in Table 2 a mean temperature balance for northwestern Europe can be derived. Again, we essentially deal with a maintenance equation. The balance consists of advection of mean temperature, heating due to dry adiabatic vertical motion, heating by horizontal eddies, and probably diabatic heating. The effect of vertical eddies seems to be small; the same applies to curvature terms.

The estimates of the various terms in Tables 1 and 2 are in good agreement with estimates by Savijärvi (1976, 1977), which are valid for a large part of the Northern Hemisphere. Assuming that vertical transport terms are small, he arrives at the same approximate balance equations for mean momentum and temperature.

The small values of vertical transports deserve more attention. Holopainen (1964) gives estimates of the transient eddy vertical momentum flux for autumn 1954 over the British Isles. For $\overline{u'\omega'}$ his kinematic and adiabatic estimates for 500 mb are -0.23 and 0.05 N m^{-1} s^{-2}, respectively. For the same level Holopainen estimates $\overline{v'\omega'}$ to be -0.92 and -0.72 N m^{-1} s^{-2} according to the two methods. These values agree fairly well with our results (see Fig. 10). As far as the convergence of vertical fluxes is concerned, we must keep in mind that the vertical velocities used in this study are derived from a model with poor vertical resolution. This means that we can describe only the smoothly varying part of the synoptic-scale ω-profile. Therefore we easily might have underestimated the convergence of, for example, $\overline{u'\omega'}$. This seems to be the case if we compare the value for $(\partial/\partial p)\overline{u'\omega'}$ ($0.5.10^{-5}$ m s^{-2}) obtained in this study with those of Holopainen. With a high-resolution in the vertical he finds convergence values of $1-4.10^{-5}$ m s^{-2} in the lower half of the troposphere. If we accept these larger values, there is not sufficient reason to neglect vertical momentum fluxes.

8. Third-order eddy statistics

In the equations for the eddy variances and covariances third-order eddy statistics appear as new variables. Van den Dool (1975) and Savijärvi (1976) have investigated third-order terms appearing in the transient eddy kinetic energy equation. They used geostrophic winds derived from NMC height-field analyses; in both studies it was concluded that third-order terms are somewhat smaller than the leading terms, but by no means negligible. Attempts to parameterize the influence of third-order terms as a diffusion of eddy kinetic energy failed. In a recent paper by Lau *et al.* (1978), the maintenance of the eddy momentum flux ($\overline{u'v'}$) is investigated. For this purpose they used NMC wind analyses. The wind analysis procedure yields essentially gradient winds in data-sparse areas. In their paper the role of third-order terms is discussed only qualitatively; third-order terms are said to be small.

Before we start the presentation of our results it is necessary to discuss whether or not third-order terms can be computed reliably. Let us look at $\overline{u'^3}$, for example. The nonzero value of this term is a result of the skewness of the probability distribution of u. It is obvious that an asymmetry in the extreme values of u' can contribute substantially

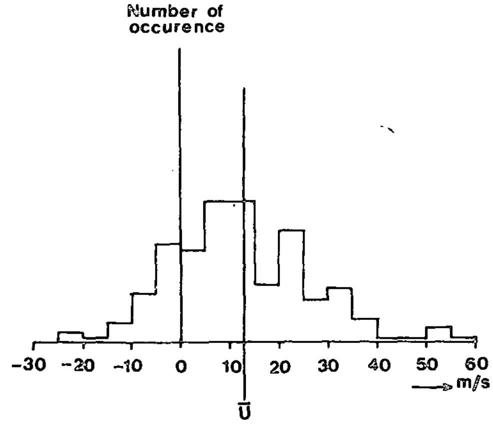

FIG. 11. Probability distribution of the zonal wind at station 1 (Sola, Norway) for 300 mb during the winter of 1976/77. Number of occurrence is given for intervals of 5 m s^{-1}.

to $\overline{u'^3}$. As an example the probability distribution of u is given for station 01415 (Sola, Southwest Norway) at 300 mb (Fig. 11). The mean value of u amounts to 12.5 m s^{-1}, while $\overline{u'^3}$ has a value of 1583 m^3 s^{-3} for this winter. It can be seen from the figure that there are about four positive extremes, which are not cancelled by negative extremes. These extremes, with a typical value of 40 m s^{-1} for u', account for almost the entire value of $\overline{u'^3}$. This strongly indicates the large uncertainty in the computed value of third-order terms for individual stations. First of all, there are sampling errors. All wind values, including the extremes, are assumed to be representative for equal intervals in time (12 h). If some of the strong winds persisted only for a few hours, we can expect a large dependence of third-order terms on the specific sampling, which is at 0000 and 1200 GMT. Second, one or two large erroneous wind measurements in the time series can obviously have great influence. This last source of errors is probably absent in third-order terms determined on the basis of geostrophic winds. The reason for this is the rejection by the objective analysis procedure of large winds which are not spatially coherent.

Fig. 12 shows the distribution of $\overline{u'^3}$ at 300 mb computed with real and with geostrophic winds. In spite of the rather noisy character in the real wind case, it can easily be seen that there is an overall predominance of positive values with a maximum over East Anglia and the North Sea. The spatial distribution of geostrophic values of $\overline{u'^3}$ shows the same general character, but the field is smoother. A comparison of the two fields also shows that the spatial gradients can be larger by a factor 2–4 if we use real winds. This is mainly due to unexpectedly large values of stations 03005 (3) and 03496 (7), which are probably due to a few large winds or wind errors. Therefore, it might be better to derive the spatial gradients from the geostrophic estimates of third-order terms. Hence in the next section, when

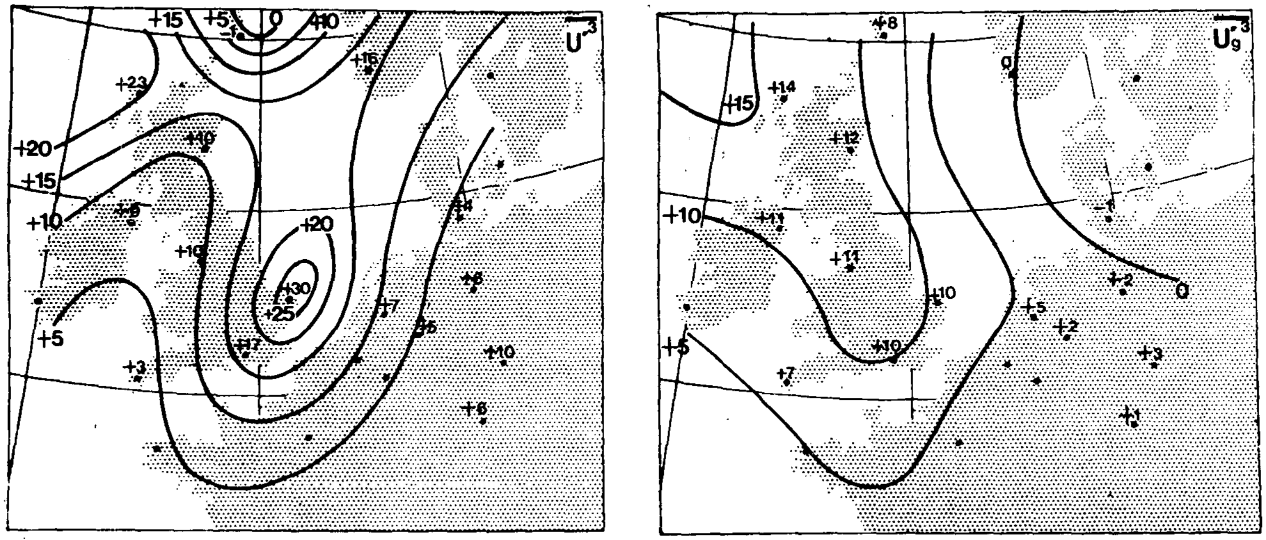

FIG. 12. Spatial distribution of $\overline{u'^3}$ at 300 mb computed with real winds (left-hand side) and with geostrophic winds (right-hand side). Units are 10^2 m^3 s^{-3}.

we discuss the order of magnitude of all terms in the eddy flux equations, gradients of third-order terms are estimated with geostrophic winds rather than real winds.

Fig. 13 displays vertical profiles of a variety of third-order terms for station 03808, together with geostrophic values at 300, 500 and 850 mb. These vertical profiles are fairly representative for the whole area. Those third-order terms that contain only velocity components are small below 500 mb; beyond this level they show a rapid increase and peak at about 300 mb. Above 300 mb a sharp decrease can be observed. This behavior is not unlike vertical profiles of the second-order terms. The third-order terms involving T' show interesting vertical profiles. The sharp peak at 300 mb is accompanied by a rapid transition to a peak of opposite sign if we cross the tropopause. This is of course related to change of sign of both $\overline{u'T'}$ and $\overline{v'T'}$ at these levels (Fig. 7).

The geostrophic estimates (crosses in Fig. 13) compare well with the profiles. It should be emphasized that at some of the other stations the deviations are much larger.

When we started this study, one of our hopes was to find an averaging period for which third-order terms would turn out to be small. If the averaging period was increased from 5 to 90 days, we found a general decrease with a factor of about 2. There was no period which could be recognized as an optimum.

Finally, concerning third-order terms containing ω, we restrict ourselves in the next section to give orders of magnitude only. We do not believe that their spatial distribution is very informative.

9. Estimates of the terms in the horizontal eddy flux equations

The estimates of the various terms in the equations for the horizontal eddy fluxes $\overline{u'^2}$, $\overline{v'^2}$, $\overline{u'v'}$, $\overline{u'T'}$ and $\overline{v'T'}$ are given in Tables 3–7. In these tables we have condensed the significance of each term into one numerical value.

In the equations for $\overline{u'^2}$ and $\overline{v'^2}$ there is an approximate balance between the advection of these quanti-

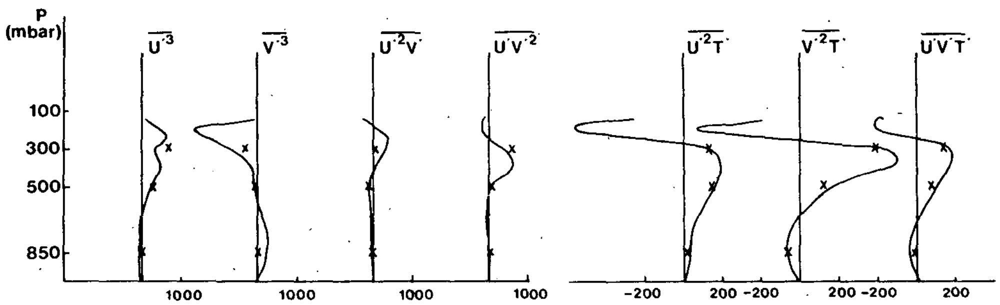

FIG. 13. Vertical profiles of third-order terms at station 9 (Camborne, England). Crosses denote geostrophic estimates. Units are m^3 s^{-3} and m^2 K s^{-2}.

ties by the mean flow, ageostrophic momentum fluxes, horizontal third-order advection and, finally, the interaction with the mean flow (see Tables 3 and 4). Terms containing vertical velocity and curvature terms turn out to be relatively small. As was the case in the mean momentum equations, the time derivatives are very small and therefore we are essentially dealing with maintenance equations, which as such have no predictive value.

Concerning the $\overline{u'v'}$ maintenance, Lau *et al.* (1978) concluded that the most important terms are the ageostrophic terms and the so-called mixing term $\overline{v'^2}(\partial\bar{u}/\partial y)$. From Table 5 it will become clear that at least for the northwestern European area a role in the balance is also played by terms other than the destruction and creation term. The approximate balance is between advection of $\overline{u'v'}$ by the mean wind, ageostrophic terms, horizontal third-order terms and interaction with the mean flow by two "mixing terms", *viz.*, $\overline{v'^2}(\partial\bar{u}/\partial y)$ and $\overline{u'^2}(\partial\bar{v}/\partial x)$. The latter term was estimated by Lau *et al.* to be at least three times smaller. Our finding is based on the fact that $\partial\bar{v}/\partial x$ can be as large as $\partial\bar{u}/\partial y$ even though it is the smaller one in general (see Fig. 14). Vertical advection terms and some of the terms describing interaction with the mean flow seem to be negligible. The same holds for many of the curvature terms, with the exception of $(\tan\varphi/a)\bar{u}(\overline{2u'^2} - \overline{v'^2})$, which is of some importance and cannot be neglected.

Table 3. As in Table 1 except for the variance of the zonal wind. Units are 10^{-5} m^2 s^{-3}.

		850	500	300	200
	$\frac{\partial}{\partial t}\frac{1}{2}\overline{u'^2}$	1	5	8	5
+	$\bar{\mathbf{V}}\cdot\nabla\frac{1}{2}\overline{u'^2}$	30	50	200	80
+	$\bar{\omega}\frac{\partial}{\partial p}\frac{1}{2}\overline{u'^2}$		8		
−	$f\,\overline{u'v'_{ag}}$	40	100	300	
−	$\overline{u'F'_x}$				
−	$\bar{u}\,\overline{u'v'}\,\frac{\tan\varphi}{a}$	1	4	10	6
−	$\bar{v}\,\overline{u'^2}\,\frac{\tan\varphi}{a}$	5	5	10	6
−	$\frac{3}{2}\overline{u'^2v'}\,\frac{\tan\varphi}{a}$	6	10	20	
+	$\overline{u'^2}\frac{\partial\bar{u}}{\partial x}$	30	40	100	50
+	$\overline{u'v'}\frac{\partial\bar{u}}{\partial y}$	10	20	40	20
+	$\overline{u'\omega'}\frac{\partial\bar{u}}{\partial p}$		4		
+	$\frac{1}{2}\frac{\partial\overline{u'^3}}{\partial x}$	40	80	100	
+	$\frac{1}{2}\frac{\partial}{\partial y}\overline{u'^2v'}$	20	40	90	
+	$\frac{1}{2}\frac{\partial}{\partial p}\overline{u'^2\omega'}$		5		
=	0				

Table 4. As in Table 1 for the variance of the meridional wind. Units are 10^{-5} m^2 s^{-3}.

		850	500	300	200
	$\frac{\partial}{\partial t}\frac{1}{2}\overline{v'^2}$	1	5	10	10
+	$\bar{\mathbf{V}}\cdot\nabla\frac{1}{2}\overline{v'^2}$	30	80	200	200
+	$\bar{\omega}\frac{\partial}{\partial p}\frac{1}{2}\overline{v'^2}$		8		
+	$f\,\overline{v'u'_{ag}}$	50	50	200	
−	$\overline{v'F'_y}$				
+	$2\bar{u}\,\overline{u'v'}\,\frac{\tan\varphi}{a}$	2	8	20	10
+	$\overline{u'^2v'}\,\frac{\tan\varphi}{a}$	4	8	10	
−	$\frac{1}{2}\overline{v'^3}\,\frac{\tan\varphi}{a}$	2	10	30	
+	$\overline{u'v'}\frac{\partial}{\partial x}\bar{v}$	6	10	40	6
+	$\overline{v'^2}\frac{\partial}{\partial y}\bar{v}$	50	100	100	60
+	$\overline{v'\omega'}\frac{\partial}{\partial p}\bar{v}$		7		
+	$\frac{1}{2}\frac{\partial}{\partial x}\overline{u'v'^2}$	10	40	100	
+	$\frac{1}{2}\frac{\partial}{\partial y}\overline{v'^3}$	20	60	100	
+	$\frac{1}{2}\frac{\partial}{\partial p}\overline{v'^2\omega'}$		10		
=	0				

In the equation for the zonal eddy temperature flux the main balance is composed of the ageostrophic term, horizontal third order terms, and two mean flow interaction terms, namely $\overline{v'T'}(\partial\bar{u}/\partial y)$ and $\overline{u'^2}(\partial\bar{T}/\partial x)$. (See Table 6.) Terms of minor importance but nonnegligible are advection $\bar{\mathbf{V}}\cdot\nabla\overline{u'T'}$, and $\overline{u'v'}(\partial\bar{T}/\partial y)$ and $\bar{\sigma}\overline{u'\omega'}$. All the other terms are at

TABLE 5. As in Table 1 except for the meridional momentum flux by large-scale eddies. Units are 10^{-5} m² s⁻³.

		850	500	300	200
	$\frac{\partial}{\partial t}\overline{u'v'}$	1	5	20	20
+	$\bar{\mathbf{V}}\cdot\nabla\overline{u'v'}$	20	60	200	200
+	$\bar{\omega}\frac{\partial}{\partial p}\overline{u'v'}$		4		
−	$f(\overline{v'v'_{ag}} - \overline{u'u'_{ag}})$	100	100	400	
−	$(\overline{v'F'_x} + \overline{u'F'_y})$				
+	$\bar{u}(2\overline{u'^2} - \overline{v'^2})\frac{\tan\varphi}{a}$	10	20	50	30
−	$\bar{v}\,\overline{u'v'}\frac{\tan\varphi}{a}$	0.6	0.8	2	0.8
+	$\overline{u'^3}\frac{\tan\varphi}{a}$	8	20	30	
−	$2\overline{u'v'^2}\frac{\tan\varphi}{a}$	2	10	30	
+	$\overline{u'v'}\frac{\partial\bar{u}}{\partial x}$	3	6	10	6
+	$\overline{v'^2}\frac{\partial\bar{u}}{\partial y}$	80	200	400	200
+	$\overline{v'\omega'}\frac{\partial\bar{u}}{\partial p}$		10		
+	$\overline{u'^2}\frac{\partial\bar{v}}{\partial x}$	50	70	300	40
+	$\overline{u'v'}\frac{\partial\bar{v}}{\partial y}$	6	10	20	6
+	$\overline{u'\omega'}\frac{\partial\bar{v}}{\partial p}$		2		
+	$\frac{\partial}{\partial x}\overline{u'^2v'}$	30	70	200	
+	$\frac{\partial}{\partial y}\overline{u'v'^2}$	20	80	200	
+	$\frac{\partial}{\partial p}\overline{u'v'\omega'}$		10		
=	0				

least one order of magnitude smaller than the leading terms.

The equation for $\overline{v'T'}$ proves to be puzzling. Recently, Wallace (1978) investigated this flux equation for the lower stratosphere. He assumed an approximate balance between $\overline{v'^2}(\partial\bar{T}/\partial y)$, $\bar{\sigma}\overline{v'\omega'}$, and $\overline{fu'_{ag}T'}$ in his attempt to explain the maintenance of countergradient heat flux from the structure of waves in the lower stratosphere. Here we must first of all make a clear distinction between the lower and upper troposphere. In the lower troposphere a balance like the one proposed by Wallace seems to be valid except that advection and third-order terms cannot be neglected altogether (see Table 7). In the upper troposphere we find $\overline{v'^2}(\partial\bar{T}/\partial y)$ to be 200 $\times$ 10^{-5}, which is much larger than any of the other computed terms. The question now arises: how is this mixing term balanced? A first counteracting term is $\overline{fu'_{ag}T'}$, for which we have computed a maximum value of 40 $\times$ 10^{-5} m K s⁻². It could be that we have underestimated this term. However, looking at the total value of $\overline{fu'T'}$, which has been computed direct from the measurements, we find values of up to 100 $\times$ 10^{-5} m K s⁻² in regions where the mixing term has its maximum. The bulk of this flux must be geostrophic. Hence, it is very unlikely that our estimate of $\overline{fu'_{ag}T'}$ is too low.

A second contributing term is $\bar{\sigma}\overline{v'\omega'}$, which also acts to destroy a positive eddy heat flux, and therefore counteracts the creation of northward flux by the "mixing term". Although the static stability at 300 mb is about two times larger than at 500 mb, the vertical momentum flux is probably smaller than at 500 mb. Hence $\bar{\sigma}\overline{v'\omega'}$ cannot be much larger than 50 $\times$ 10^{-5} m K s⁻². This means that there must be a third contributing term with a value of the order of 100 $\times$ 10^{-5} m K s⁻².

A candidate which was not numerically evaluated is $\overline{v'Q'}/c_p$. How large can this term be? We can try to answer this question from our knowledge that v and ω are correlated ($\rho \approx -0.5$). Therefore, southerly flow seems to be connected with rising motion and, consequently, clouds in a thick layer of the troposphere. It is well known that in the layer above the clouds heat is rapidly lost due to an

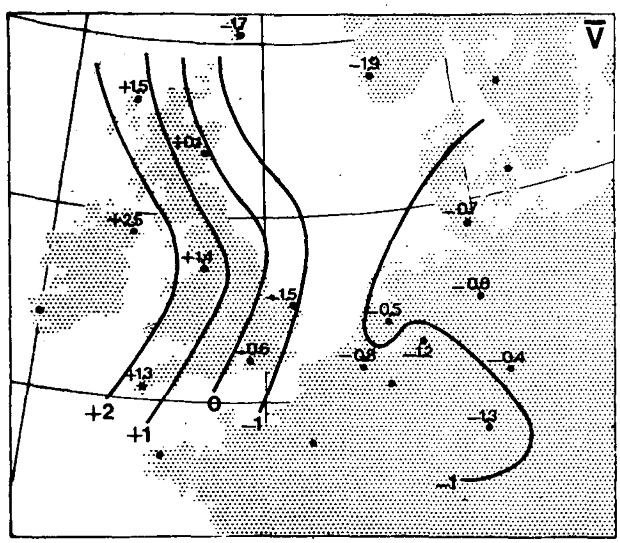

FIG. 14. Spatial distribution of the mean meridional wind at 300 mb. Units are m s⁻¹. Note the large gradients in zonal direction.

increase of outgoing IR radiation. According to Paltridge and Platt (1976) the associated cooling may amount to several degrees per hour. If we accept 10 K day^{-1} as a typical value for a rather thick layer above the clouds, then Q'/c_p is about 10×10^{-5} K s^{-1}. Combined with a typical value for

TABLE 6. As in Table 1 except for the zonal temperature flux by large-scale eddies. Units are 10^{-5} m K s^{-2}.

		850	500	300	200
	$\frac{\partial}{\partial t}\overline{u'T'}$	0.5	1	2	2
+	$\bar{\mathbf{V}}\cdot\nabla\overline{u'T'}$	5	10	30	30
+	$\bar{\omega}\frac{\partial}{\partial p}\overline{u'T'}$		0.8		
−	$f\overline{T'v'_{ag}}$	20	40	60	
−	$\overline{T'F'_x}$				
−	$\frac{\overline{u'Q'}}{c_p}$				
−	$\bar{u}\,\overline{v'T'}\frac{\tan\varphi}{a}$	2	2	5	10
−	$\bar{v}\,\overline{u'T'}\frac{\tan\varphi}{a}$	0.9	0.7	0.8	0.8
−	$\overline{2u'v'T'}\frac{\tan\varphi}{a}$	1	3	6	
+	$\overline{u'T'}\frac{\partial\bar{u}}{\partial x}$	5	5	6	6
+	$\overline{v'T'}\frac{\partial\bar{u}}{\partial y}$	20	10	20	40
+	$\overline{\omega'T'}\frac{\partial\bar{u}}{\partial p}$		2		
+	$\overline{u'^2}\frac{\partial\bar{T}}{\partial x}$	20	10	60	40
+	$\overline{u'v'}\frac{\partial\bar{T}}{\partial y}$	5	10	20	8
−	$\overline{u'\omega'}\left(\frac{R\bar{T}}{pc_p}-\frac{\partial\bar{T}}{\partial p}\right)$		8		
−	$\frac{R}{pc_p}\bar{\omega}\,\overline{u'T'}$		0.4		
+	$\frac{\partial}{\partial x}\overline{u'^2T'}$	7	40	40	
+	$\frac{\partial}{\partial y}\overline{u'v'T'}$	10	20	20	
+	$\frac{\partial}{\partial p}\overline{u'\omega'T'}$		2		
−	$\frac{R}{pc_p}\overline{u'\omega'T'}$		0.1		
=	0				

TABLE 7. As in Table 1 except for the meridional temperature flux by large-scale eddies. Units are 10^{-5} m K s^{-2}.

		850	500	300	200
	$\frac{\partial}{\partial t}\overline{v'T'}$	0.8	2	3	3
+	$\bar{\mathbf{V}}\cdot\nabla\overline{v'T'}$	10	20	30	30
+	$\bar{\omega}\frac{\partial}{\partial p}\overline{v'T'}$		2		
+	$f\overline{T'u'_{ag}}$	40	40	40	
−	$\overline{T'F'_y}$				
−	$\frac{\overline{v'Q'}}{c_p}$				
+	$2\bar{u}\,\overline{u'T'}\frac{\tan\varphi}{a}$	3	7	10	10
+	$\overline{u'^2T'}\frac{\tan\varphi}{a}$	1	4	6	
−	$\overline{v'^2T'}\frac{\tan\varphi}{a}$	1	6	10	
+	$\overline{u'T'}\frac{\partial\bar{v}}{\partial x}$	9	10	20	6
+	$\overline{v'T'}\frac{\partial\bar{v}}{\partial y}$	9	6	6	10
+	$\overline{\omega'T'}\frac{\partial\bar{v}}{\partial p}$		1		
+	$\overline{u'v'}\frac{\partial\bar{T}}{\partial x}$	2	2	8	6
+	$\overline{v'^2}\frac{\partial\bar{T}}{\partial y}$	40	90	200	80
−	$\overline{v'\omega'}\left(\frac{R\bar{T}}{pc_p}-\frac{\partial\bar{T}}{\partial p}\right)$		30		
−	$\frac{R}{pc_p}\bar{\omega}\,\overline{v'T'}$		0.2		
+	$\frac{\partial}{\partial x}\overline{u'v'T'}$	10	20	20	
+	$\frac{\partial}{\partial y}\overline{v'^2T'}$	10	30	30	
+	$\frac{\partial}{\partial p}\overline{v'\omega'T'}$		1		
−	$\frac{R}{pc_p}\overline{v'\omega'T'}$		0.1		
=	0				

v', say, 15 m s^{-1}, it seems that $\overline{v'Q'}/c_p$ can be of the order of 100×10^{-5} m K s^{-2}. Therefore, this diabatic heating term is important and should be taken into account.

Perhaps we also have the key to the explanation of the countergradient eddy heat fluxes at 300 mb over northwestern Europe, whereas in the zonal average this flux is small but downgradient. At every level in the troposphere the mixing term tries to establish a poleward heat flux. The diabatic heating term $\overline{v'Q'}/c_p$ is important everywhere where v' is correlated to ω'. Above the cloud layer the diabatic term tends to counteract the northward heat flux, which may reverse sign if the cloud layer is sufficiently thick. It is not impossible that cloud layers over northwestern Europe have a considerable vertical extent, because the air that rises in the cyclones is warmer (see Fig. 6) and probably moister than in the zonal average.

Concerning the lower stratosphere, represented here by 200 mb, it is difficult to draw any conclusion. The mixing term becomes smaller because the temperature gradient has vanished. Because we have no vertical velocities and ageostrophic terms for 200 mb, we cannot examine the balance.

To summarize the results of the five flux equations the following overall picture can be composed. *Important terms* are advection by the mean flow, ageostrophic processes, some of the many mean flow interaction terms, especially the mixing terms that try to produce downgradient fluxes and horizontal third-order advection terms. *Small terms* are the time derivatives of the fluxes, curvature terms and all processes depending on vertical velocities except $\bar{\sigma}\overline{v'\omega'}$ and $\bar{\sigma}\overline{u'\omega'}$.

10. Discussion and concluding remarks

In the introduction we have explained that the purpose of this study is to investigate possibilities of making long-range weather forecasts. More specifically, we consider the question of whether or not one can predict the anomalies in monthly or seasonal mean circulation patterns with an SD model. In order to derive model equations we have scaled the original equations that describe the evolution of the time-mean state of the atmosphere. Closure of these equations has to be accomplished somehow. By scaling the equations for the evolution of the horizontal second-order eddy statistics we have tried to derive these closure relations.

The results of the scaling are given in Tables 3–7. We shall not repeat all details but only summarize and discuss the most important features.

First, we can conclude that starting from initial mean conditions an integration in time towards the next month or season is completely useless. This holds for every equation investigated in the previous sections. The reason is that the time derivatives are negligible compared, for example, to advection, and the system therefore loses its prognostic ability. This does not mean, however, that general circulation experiments with a time-dependent SD model are also useless (Kurihara, 1970); in this kind of experiment we only have to parameterize the eddies in such a way that the resulting model atmosphere behaves well in a statistical sense. The path in the phase space is realistic but not applicable to the development of the real atmosphere. The above conclusion seems at first sight to imply that time-averaged SD models cannot be used for prediction. There is a way out, however. At best these models can be used to describe stationary solutions which result from known internal or external forcing. This makes sense only if the variability of monthly or seasonal mean patterns is not completely caused by day-to-day weather fluctuations.

Simple closure relations for the horizontal eddy fluxes cannot be derived from eddy flux equations. Scaling arguments show that new unknowns in these eddy flux equations are large and therefore the set of equations has again to be closed in a, probably, more complicated way. The first reason is that third-order terms are large and cannot simply be neglected. They are determined by a few extremes in the wind field. Because these extremes are deterministically unpredictable, it is probably impossible to find parameterization relations. Hence the existence of extremes is a limiting factor in long-range predictability.

A second reason for difficulties with the closure scheme is the ageostrophic terms in the eddy flux equations. They are in almost every case the dominant terms and it is not clear at all how to deal with them in an SD model. The foregoing implies that the possibility of parameterizing second-order eddy statistics directly should be explored in greater depth.

Vertical fluxes are small, in general, with the exception of $\bar{\sigma}\bar{\omega}$ in the equation for $\bar{\bar{T}}$ and $\bar{\sigma}\overline{u'\omega'}$ and $\bar{\sigma}\overline{v'\omega'}$ in the equations for $\overline{u'T'}$ and $\overline{v'T'}$.

The equation for $\overline{v'T'}$ proves to be puzzling. In the upper troposphere it was found that the mixing term $\overline{v'^2}(\partial T/\partial y)$ is much larger than any of the other investigated terms. Although we did not compute the diabatic term $\overline{v'Q'}/c_p$, it seems that this must be an important counteracting term at these levels. This diabatic term is probably responsible for the occurrence of countergradient eddy heat fluxes over northwestern Europe at 300 mb.

For this study we used about 15 radiosonde stations for one winter and for a small spot in the atmosphere. Can we derive any general conclusion from such a limited attack of the problem? Of course, many of the features arrived at in this paper are valid only in this small area. But each SD model must be able to describe these features for, among other things, this particular area. Besides, we are convinced that much of the scaling presented in Tables 3–7 is valid for a great part of the extratropics. The fact that we used one winter only is not a serious objection. It is not only important to know, for instance, whether third-order terms are climatologically significant, it is more meaningful to know how important third-order terms are for one season or one month. After all, that is what we want to predict—one individual case.

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APPENDIX

List of Symbols

x, y	curvilinear coordinates pointing east and north
λ, φ	geographical longitude and latitude
dx	$a \cos\varphi d\lambda$
dy	$a d\varphi$
p	pressure, vertical coordinate
t	time
u	zonal wind component $[=dx/dt]$
v	meridional wind component $[=dy/dt]$
ω	vertical wind component $[=dp/dt]$
u_g	geostrophic zonal wind component $[= -(\alpha/f)(\partial p/\partial y)]$
u_{ag}	ageostrophic zonal wind component $[=u - u_g]$
v_g	geostrophic meridional wind component $[=(\alpha/f)(\partial p/\partial x)]$
v_{ag}	ageostrophic meridional wind component $[=v - v_g]$
T	temperature
z	height above sea level
Φ	geopotential $[=gz]$
α	specific volume
a	average radius of the earth
c_p	specific heat at constant pressure
f	Coriolis parameter $[=2\Omega \sin\varphi]$
g	acceleration of gravity
R	gas constant
F_x, F_y	components of horizontal friction
Q	diabatic heating
Ω	angular velocity of the earth
$\bar{F}$	time average of F
F'	departure of time average of F $[=F - \bar{F}]$
σ	static stability $[=(RT/pc_p) - (\partial T/\partial p)]$

REFERENCES

Adem, J., 1964: On the physical basis for the numerical prediction of monthly and seasonal temperatures in the troposphere-ocean-continent system. *Mon. Wea. Rev.*, **92**, 91–103.

——, 1970: On the prediction of mean monthly ocean temperatures. *Tellus*, **22**, 410–430.

Dool, H. M. van den, 1975: Three studies on spectral structures of the horizontal atmospheric motion in the time domain. Ph.D. thesis, University of Utrecht, 128 pp.

Hantel, M., 1976: On the vertical eddy transports in the Northern Hemisphere—1. Vertical eddy heat transport for summer and winter. *J. Geophys. Res.*, **81**, 1577–1588.

——, and J. M. Hacker, 1978: On the vertical eddy transports in the Northern Hemisphere—2. Vertical eddy momentum transport for summer and winter. *J. Geophys. Res.*, **83**, 1305–1318.

Holopainen, E. O., 1964: Investigation of friction and diabatic processes in the atmosphere, Pap. No. 101, Dept. of Meteorology, University of Helsinki, 45 pp.

——, 1973: An attempt to determine the effects of turbulent friction in the upper troposphere from the balance requirements of the large-scale flow: A frustrating experiment. *Geophysika*, **12**, 151–176.

Kurihara, Y., 1970: A statistical-dynamical model of the general circulation of the atmosphere. *J. Atmos. Sci.*, **27**, 847–870.

Lau, N. G., H. Tennekes and J. M. Wallace, 1978: Maintenance of the momentum flux by transient eddies in the upper troposphere. *J. Atmos. Sci.*, **35**, 139–147.

——, 1978: On the three-dimensional structure of the observed transient eddy statistics of the Northern Hemisphere wintertime circulation. *J. Atmos. Sci.*, **35**, 1900–1923.

Lorenz, E. N., 1967: The nature and theory of the general circulation of the atmosphere. WMO-N.218.TP.115, 161 pp.

——, 1969: The predictability of a flow which possesses many scales of motion. *Tellus*, **21**, 289–307.

Oort, A. H., and E. M. Rasmusson, 1971: Atmospheric circulation statistics. NOAA Professional Paper 5, 323 pp.

Palmén, E., and C. W. Newton, 1969: *Atmospheric Circulation Systems. International Geophysics Series*, Vol. 13, Academic Press, 603 pp.

Paltridge, G. W., and C. M. R. Platt, 1976: Radiative processes in meteorology and climatology. *Developments in Atmospheric Science*, Vol. 5, Elsevier, 318 pp.

Saltzmann, B., R. M. Guttoso and A. Fleisher, 1961: The meridional eddy transport of kinetic energy at 500 mbar. *Tellus*, **13**, 293–295.

Savijärvi, H., 1976: The interaction of the monthly mean flow and large-scale transient eddies in two different circulation types. Part 1: The forcing effect of large-scale turbulence and kinetic energy balance. *Geophysika*, **14**, 23–46.

——, 1977: The interaction of the monthly mean flow and large-scale transient eddies in two different circulation types. Part 2: Vorticity and temperature balance. *Geophysika*, **15**, 207–230.

Vernekar, A. D., and H. D. Chang, 1978: A statistical-dynamical model for stationary perturbations in the atmosphere. *J. Atmos. Sci.*, **35**, 433–444.

Wallace, J. M., 1978: Trajectory slopes, countergradient heat fluxes and mixing by lower stratosphere waves. *J. Atmos. Sci.*, **35**, 554–558.