An Efficient and Accurate Method of Continuous Time Interpolation of Large-Scale Atmospheric Fields

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ABSTRACT

A method for time interpolation based on the climatological speed of large-scale atmospheric waves that is empirically determined is proposed. When tested on a 7-yr dataset this method is found to be easy to use, has good accuracy, and is, in fact, considerably more accurate than the much used linear time interpolation. The gain in accuracy is particularly large for mobile synoptic waves. Several applications of a time-continuous description of the atmosphere are discussed.

1. Introduction

In meteorological data studies one is often faced with a "missing data" problem. Depending on the purpose of the research, one can sometimes ignore this problem, but very often researchers feel forced to design a workable recipe to replace missing data, particularly when a complete time series sampled at regular intervals is needed. Filling in the missings is commonly done by a previous or future value, a climatological value, (linear) interpolation between nonmissing neighbors in time, etc. Among these, linear time interpolation (LTI) is the most frequently used and probably considered acceptable for most purposes. Below LTI is the benchmark to which the featured method will be compared.

This paper applies most to a situation when one has a time series of maps (analyses or forecasts) and one (or more) of the maps is missing or considered suspect, or, more generally, when one needs to know the state of the atmosphere at some in-between time.

The method presented here is based on the simple fact that zonal (or spherical) harmonics of anomaly values of a meteorological variable have a definable climatological zonal phase speed associated with them. This phase speed can be defined and determined empirically from a multiyear dataset by employing the so-called phase-shifting method first described in Cai and Van den Dool (1991). When applied, for example, to 500-mb height data, one will obtain the results reported in Table 1, where we show the climatological phase speed (as a function of latitude) for zonal harmonics in 500-mb height anomalies based on once-daily data during December–March 1987–93.

From Table 1, one can see that zonal harmonics of 500-mb height anomalies move climatologically according to a phase speed versus zonal wavenumber m relationship akin to the Rossby equation (Van den Dool and Cai 1994), that is, long waves move westward, particularly at low latitudes and short waves move eastward, particularly in middle to high latitudes. Note, however, that the speeds are derived in purely empirical fashion; a theoretical Rossby equation or even the vorticity equation has not been invoked at all. The recipe to calculate the phase speeds is given in the appendix.

The above does not necessarily imply that periodic waves move around as physical entities. Rather, the sine and cosine base functions, although arbitrary, are rather efficient for this purpose. Because Rossby et al. (1939) used the same base functions, and because heights represent the rotational part of the atmospheric flow quite well, the famous Rossby wave speed vs wavenumber relationship is quite apparent from Table 1.)

Once the phase speeds in Table 1 have been determined, it is trivial to make "forecasts." This is done by decomposing the height anomaly field at the initial time (when data are available!) into zonal harmonics and applying the speeds in Table 1 at each latitude for the desired duration: 6, 12, or any number of hours. We will call this a forecast based on empirical wave propagation (EWP). In this paper, there is no assumed change in wave amplitude; there is change only due to zonal motion of each harmonic. We have found that the motion of the waves can be relied upon to such an extent that EWP, as a forecast, is better on each and every day than persistence. This is particularly so in the

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TABLE 1. Empirically determined west-to-east phase speed (m s^{-1}) of zonal harmonics in 500-mb height anomalies as a function of latitude and wavenumber, m.

	m									
Latitude	1	2	3	4	5	6	7	8	9	10
60°	−4.2	−0.1	1.4	2.9	5.0	6.9	7.0	6.9	6.0	7.2
50°	−3.7	−0.9	0.3	2.5	5.1	6.3	8.1	9.2	9.6	9.2
40°	−3.0	0.6	0.6	2.6	4.9	6.6	8.3	9.5	9.8	9.6
30°	−5.6	0.3	−0.2	0.3	1.1	3.9	6.1	6.9	7.7	8.0
20°	−13.3	−4.3	−4.2	−3.3	−1.2	1.9	2.8	3.2	3.5	3.6
10°	−21.0	−8.3	−7.4	−5.4	−4.0	−2.3	−1.7	−1.7	−0.9	0.0
0°	−24.1	−7.4	−3.2	−2.4	−1.4	1.1	0.7	1.4	1.3	0.0
−10°	−26.5	−9.0	−7.9	−6.7	−6.0	−4.9	−3.8	−3.2	−2.5	−1.7
−20°	−21.1	−8.2	−7.0	−6.7	−4.8	−2.7	−2.0	−1.0	−0.8	−0.5
−30°	−12.0	−3.4	−4.3	−3.0	−0.8	1.4	2.3	2.5	3.4	4.0
−40°	−8.6	−0.8	−2.4	0.9	4.9	7.9	9.3	8.6	8.5	8.7
−50°	−5.7	−1.2	0.6	3.6	7.0	10.0	12.2	13.6	14.4	15.0
−60°	−1.2	3.0	3.4	5.1	6.6	9.0	10.2	11.5	13.3	13.8

Season December–March period 1987–93.

Southern Hemisphere where synoptic wave speeds are high, and persistence becomes a bad forecast for short-scale waves within hours.

As described above, it is natural to go forward in time (denoted EWP$^+$) but since the method is reversible, an integration backward in time (EWP$^-$) is equally reasonable. There is no reason as to why EWP$^-$ would be less skillful than EWP$^+$ for the same amount of lapsed time. The proposed time interpolation scheme then is to apply EWP$^+$ and EWP$^-$ to a previous and a future field, respectively, and to take a weighted average at the time where a missing (or bad) field needs replacement. The weights would be equal and equal to 0.5 when, as is often the case, the missing data point is exactly in the middle of available data.

Strictly speaking, EWP is an extrapolation, and we propose to average two extrapolations, one forward and one backward, which, for lack of a better word, we call an interpolation. No formal root-mean-square error minimization of any kind is used. The phase-shifting method underlying EWP is entirely intuitive. Other attributes to describe EWP would be quasi-Lagrangian (following waves) and spectral.

The proposed EWP interpolation (EWPI) scheme is not only applicable to the missing data problem, but more generally to any situation in which one needs meteorological fields at a particular time. EWPI provides a time-continuous description of the large-scale field. Often the results of forecasts and analyses produced by a numerical weather prediction (NWP) model are made available to the users at 12- or 24-h intervals only. EWPI can be called upon, at the user end, for rather reliable interpolation for any variable at any level to any time in between.

One particular context is where data from a global model or analysis, available at certain 6-, 12-, or 24-h intervals only, are used to provide lateral boundary conditions for a high-resolution limited-area model. Somehow the boundary conditions need to be interpolated in time to accommodate the time stepping of the nested model, and in this context LTI is common (Giorgi et al. 1994).

Similarly, a data quality control program in conjunction with a data assimilation system often requires a "guess" at an off-hour, that is, at the time the observations happen to be taken. Especially when this is retroactive, as in a climate data assimilation system (Kalnay and Jenne 1991), EWPI could be applied to provide an "in-between guess" based on previous and future analyses.

Another context is one in which a user has a seemingly complete dataset, but has to check whether all fields are reasonable or in the right time order. This is often done by comparing each datum to its climatological mean, or by calculating the rms difference between successive maps. EWPI can be used as a more strict monitoring system to spot those outliers that might otherwise go unnoticed.

In diagnostics studies there is often a need for an estimate of the time derivative (Brunet et al. 1995) to study the full balance of an equation. We suggest that EWPI can be used to obtain an estimate of the time derivative that is superior to the traditional time derivative calculated from the difference between successive daily or 12-h data.

In section 2 we present a demonstration of the results of EWPI applied to presumed missing 500-mb height data. Conclusions and discussion are in section 3.

2. Demonstration

In this study, we use a dataset of daily global 500-mb height analyses for seven successive Northern Hemisphere (NH) winters (16 December–15 March),

TABLE 2. Skill of replacing missing 500-mb height data along 50°N and 50°S by various methods.

	AC—50°N	AC—50°S
Persist 24 h forward	0.734	0.640
EWP^+ 24 h forward	0.812	0.833
LTI	0.845	0.779
EWPI = (EWP^+ + EWP)/2.	0.913	0.932

Skill is measured here by the anomaly correlation and is based on 602 cases in seven winters.

produced in real time at NCEP (aka NMC) during 1987–93. For each day, the climatology valid at that day is subtracted, leaving daily anomaly fields. Each of the 602 fields in turn is assumed missing (in reality data are present!). We then apply EWPI = (EWP^+ + EWP^-)/2 and verify the result against the analysis that was assumed missing and compare the result to LTI. Results are accumulated for all days in the seven seasons along 50° of both hemispheres. Because the data are at daily intervals, EWP forward/backward is applied for a duration of 24 h starting from data 48 h apart so as to reach the midpoint. To be as realistic as possible the wavespeeds are applied in a ''cross validation'' mode, that is, when doing the interpolations for winter 1990–91, for example, the phase speeds are based on data in the six other winters only. Shown in Table 2 are anomaly correlations (AC) based on 20 waves (and the zonal mean anomaly, which is persisted) collectively. The AC is defined as in Saha and Van den Dool (1988)—usually the AC is for a spatial domain but in Table 2 the summation in space is just over all grid points along 50°N or 50°S, at 2.5° distance.

Table 2 shows rather convincingly that EWP forward (or backward, not shown, but equal in skill to the second decimal) is a considerably better guess than persistence in both hemispheres. The difference is nearly 0.20 in the Southern Hemisphere (SH), where waves move fast. Likewise, compared to LTI, EWPI gains 0.07 and 0.15 in the Northern and Southern Hemispheres, respectively. The scores for EWPI are in excess of 0.90, thus making a very reliable substitute for missing data.

Note also from Table 2 that LTI is barely an improvement over EWP^+ (in fact, LTI is worse than EWP^+ in the SH), even though LTI uses future information and EWP^+ does not.

The results are further analyzed and explained by breaking down the above scores by wavenumber. Shown in Table 3 are the ACs (×100) for single waves based on 602 interpolations along 50°S. It is seen that EWPI gains enormously over LTI in the eastward-moving short waves. This is not just a gain, but a veritable rescue mission in which systematically bad interpolations (highly negative AC) are turned into worthwhile estimates. We have accidentally highlighted a very bad (but obvious) property of linear interpolation. LTI and persistence place certain waves in exactly the wrong phase, for a certain combination of wave speed and temporal sampling. For example, zonal wavenumber 9 moves about 180° eastward in a day. Both LTI and persistence ignore this knowable aspect and feature disastrous estimates for these waves. The time mean amplitude of the anomaly waves decreases only slowly with wavenumber, so the correct treatment of waves 6–12 is important. For even higher wavenumbers, the results are bad by all methods, probably because the amplitudes (only a few geopotential meters) are small and in the noise level of the analysis and observational network.

The results would be the same qualitatively, but somewhat different quantitatively, if data spaced at 12- or 6-h intervals was used for this demonstration. For instance, when data are available every 12 h and assuming missings as before, the scores are higher overall, the gain of EWPI over LTI is somewhat less, and the wavenumber range where LTI performs very poorly will start at higher wavenumbers, that is, have less amplitude.

From Table 3 it can be seen that nearly all of the gains of EWPI over LTI are in the synoptic scale waves. Considering Table 1, one might have thought that the motion of the long waves (3–6 m s^{-1} westward) would also have its payoff. However, for the AC, only movement in terms of wavelength counts. For instance, a speed of 7.5 m s^{-1} implies about a quarter wavelength displacement for $m = 10$ in 1 day (thus outperforming persistence handily), while for $m = 1$ the displacement (for the same speed) is only one-tenth of a quarter wavelength. In general it can be said that, relative to LTI, EWPI is worth the investment only when wave displacements (in terms of their own wave-

TABLE 3. Skill of replacing missing 500-mb height data along 50°S by LTI and EWPI.

m	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	0–20
LTI	97	94	95	95	94	89	50	−27	−54	−63	−53	−43	−28	−12	−13	−13	78
EWPI	97	94	95	95	96	96	94	89	79	69	52	42	29	16	17	20	93

Skill is measured here by the anomaly correlation (multiplied by 100) as a function of zonal wavenumber *m* and is based on 602 cases in seven winters.

length) are considerable. It is hard to find a latitude or variable or level where EWPI's gain over LTI is much larger than that in the example shown for 50°S for 500-mb heights. On the other hand, most variables at most levels along 50°S (including surface pressure) feature phase speeds comparable to those in Table 1; therefore, the examples shown are not atypical.

With reference to Table 3, it is also noteworthy that wave 6, which moves at a speed of 10 m s^{-1} in the SH, can be determined almost as accurately as the slow longer waves (AC > 0.90), whereas $m = 8-11$ feature much lower AC. One has to remember that the speeds in Table 1 are "climatological" and are not necessarily close to the speed for the wave on any given day. Apparently, $m = 6$ moves at a relatively constant and thus predictable speed. Wave speed for $m = 8-11$ on the other hand, while climatologically fast (15 m s^{-1}) and progressive, varies considerably from day to day.

For visualization purposes we calculated the AC of EWPI and LTI for every single day on the domain 30°–80°N. The frequency distribution of the AC is shown in Fig. 1. It is clear that EWPI is considerably better than LTI, because the entire distribution has shifted to the right and is fairly narrow. In fact, on more than 10% of the days, the AC for EWPI is in excess of 0.96, and cases worse than AC = 0.88 are rare. We then picked a single case (24 January 1989) where the ACs (on 30°–80°N) for EWPI and LTI were 0.93 and 0.72, respectively; a difference somewhat larger than average, because the score of LTI for this case is about as low as it can get (see Fig. 1). Shown in Fig. 2 are the 500-mb height anomaly fields observed on 24 January 1989 (a), due to EWPI (b), and LTI (c). The latter two used data from January 23 and 25. Panels (d) and (e) feature the error EWPI-OBS and LTI-OBS. By comparing (d) and (e) it is clear that there are wave trains in the Pacific and Atlantic whose forward propagation is treated very well by EWPI and poorly by LTI. Generally, LTI has a suppressed magnitude [see panels (c) and (e)].

Finally, we calculated a local anomaly correlation based on 600+ pairs of observed and interpolated anomaly values at each location. A map of the spatial distribution of this AC is shown in Fig. 3 for EWPI (top) and LTI (bottom). It is clear that EWPI is a satisfactory method at almost all locations, AC < 0.9 occurring only in some small areas over the Atlantic and the Black Sea. As we have discussed before, LTI has the largest problems where and when synoptic systems are common. It is not too surprising that the AC for LTI is as low as 0.7–0.8 in the storm tracks off the east coasts of the two continents, and EWPI makes for large improvements in these areas. Note, however, that the EWPI map still has a weak minimum in the Atlantic, coinciding with the much stronger minimum in the LTI map, that is, the wave speeds applied in EWP (which are longitude independent by construction) may not be as accurate for the Atlantic Basin as they are elsewhere, the Pacific in particular. Outside the storm tracks, the gain of EWPI over LTI is smaller, particularly where the latter already reaches AC > 0.95 north of Alaska and eastern Siberia.

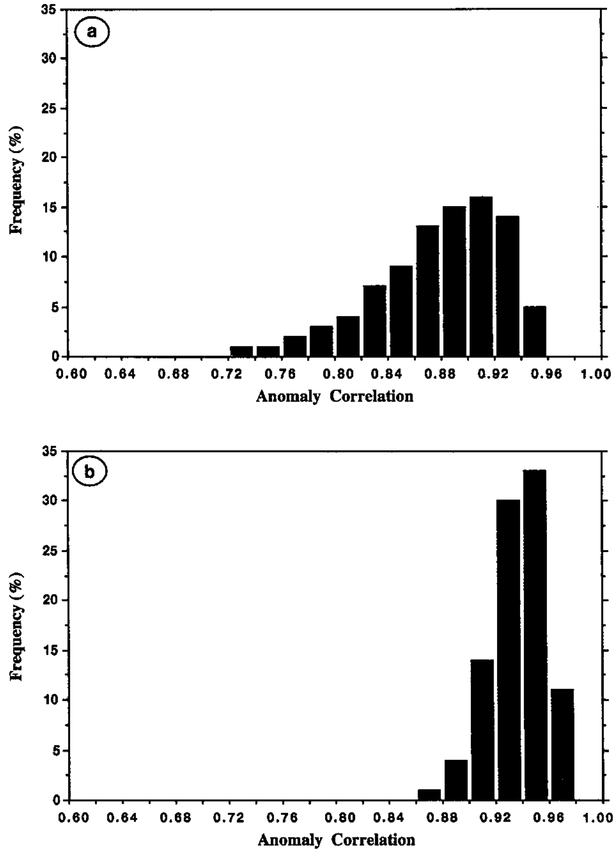

FIG. 1. The frequency distribution of the AC on the domain 30°–80°N for methods (a) LTI and (b) EWPI. The width of the histograms is 0.02.

3. Conclusions, discussion, and application

We have shown that an interpolation method (EWPI) based on the empirically determinable motion of large-scale atmospheric waves has good accuracy, and is, in fact, more accurate than the much used LTI. The gain in accuracy is particularly large for synoptic-scale waves.

To apply EWPI, some investment has to be made of course. A dataset covering a few years at least must be assembled and must be amenable to spectral transform. Also, a climatology needs to be available to form the anomaly part of the fields. In our experience it takes a minimum of 5 years to obtain reliable results for seasonally pooled daily data. Phase speeds for the parameter, level, and season of interest can then be determined and used henceforth on independent data. (To

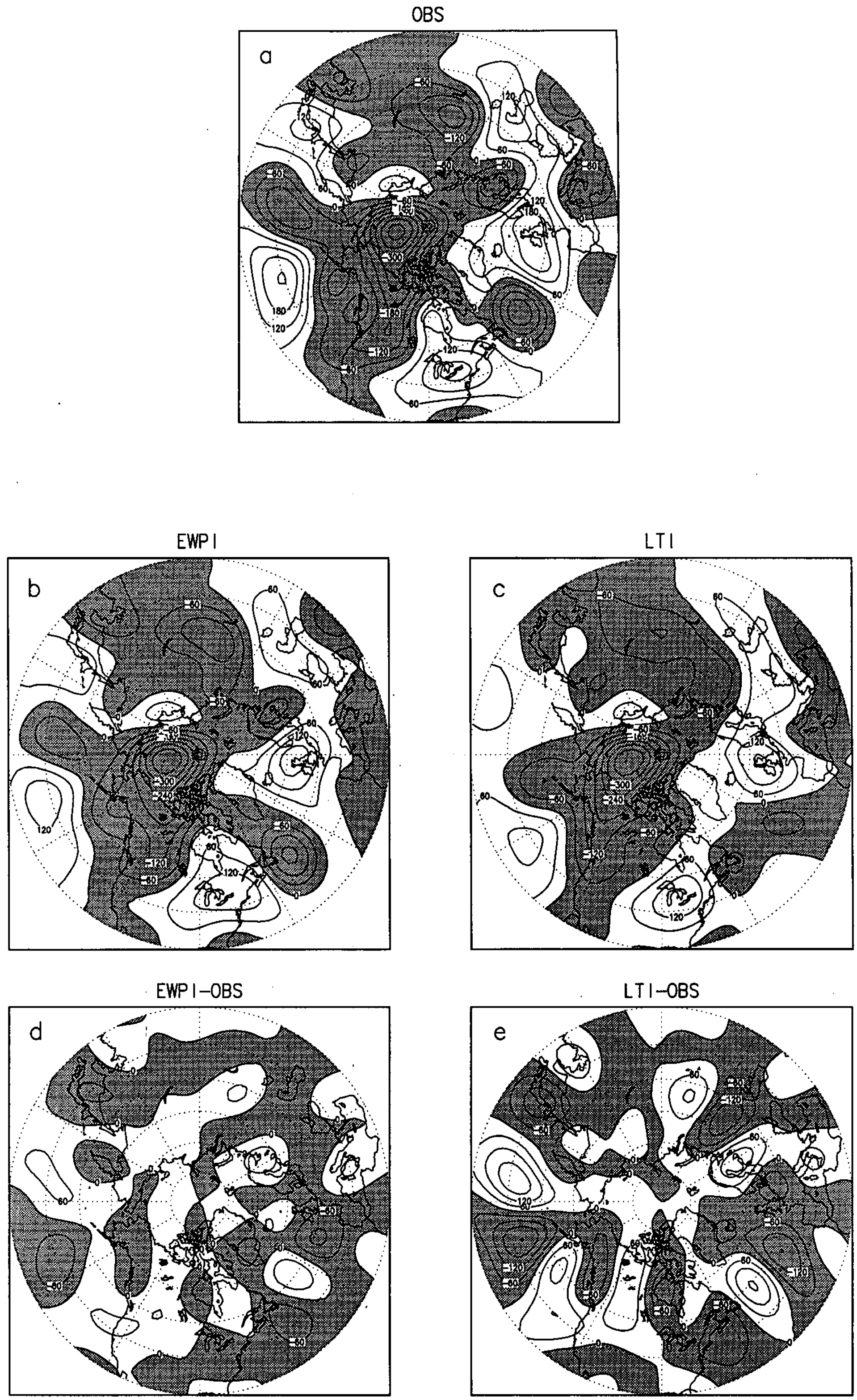

FIG. 2. The 500-mb height anomalies at 60-gpm contours (negative shaded) at valid time 24 January 1989. (a) Observed; (b) EWPI; (c) LTI. Panels (d) and (e) show EWPI-OBS and LTI-OBS, respectively.

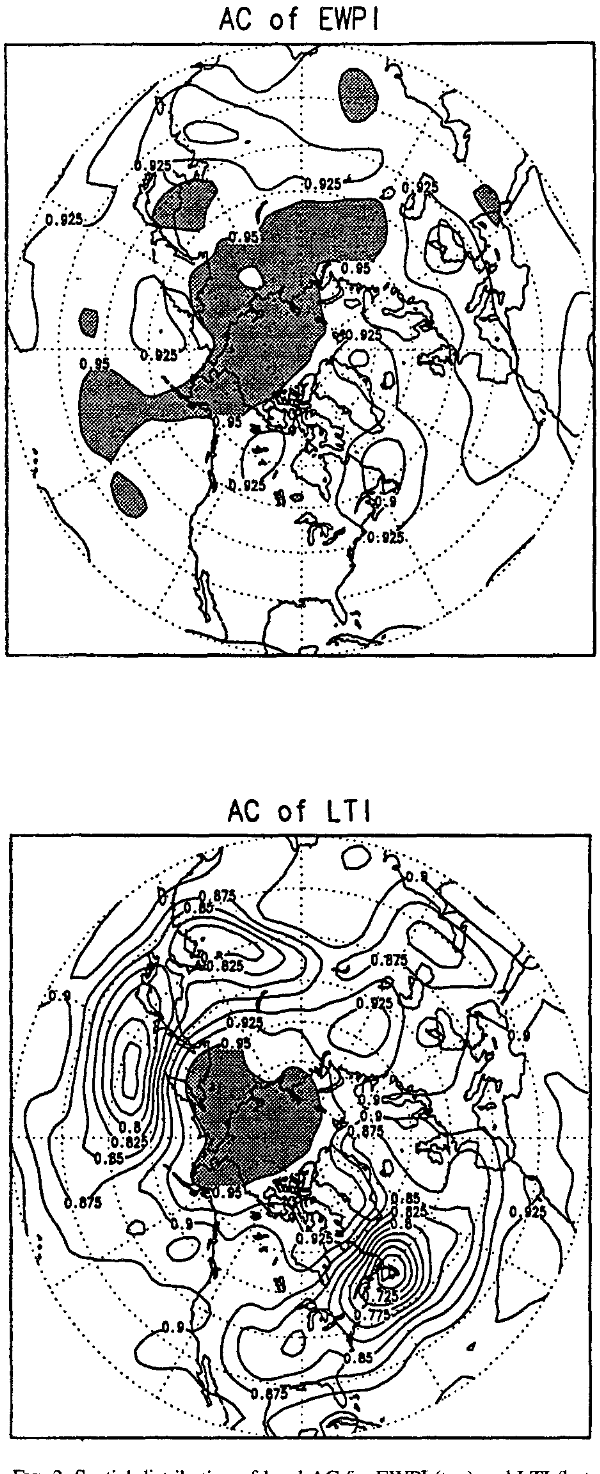

FIG. 3. Spatial distribution of local AC for EWPI (top) and LTI (bottom). Values for AC > 0.95 are shaded. Contour interval is 0.025.

determine phase speeds based on daily data by calendar month would require probably 15 years of data.) The calculations can be done following the details given in the appendix.

EWPI is applied here to anomalies, and not to the total field. For EWPI to work well, it is most important to decompose the total fields into components with speeds that differ maximally. Step one is to recognize that the climate is almost constant, while anomalies assume larger time derivatives. The anomalies are further decomposed into waves that move westward and eastward.

In section 2 we used the ever popular 500-mb height data. While EWP$^+$ is not as accurate as a barotropic model for 500-mb height forecasts (Qin and Van den Dool 1996), it should be pointed out that EWP$^+$ (EWPI) can be applied to any variable at any level with good gains in accuracy over persistence (LTI). A barotropic model is restricted to the midlatitude midtroposphere, and rotational quantities only. There are no such restrictions for EWP. We present an example of another field at another level: the velocity potential (VP) at 200 mb at 50°N. The speeds for waves 1–10 are recorded to be 6.5, 5.6, 5.4, 6.0, 7.0, 8.1, 9.4, 10.5, 10.8, and 11.4 m s^{-1}, respectively. Note that the long waves in the VP field move in a direction opposite to the geopotential height. Applying, as before, LTI and EWPI to the VP field, we find AC = 0.812 and 0.847, respectively. Further analysis reveals the same reasons as before for the improvement, that is, the synoptic waves in VP are treated much better by EWPI. However, because the VP is much more dominated by the largest scales, the overall improvement in AC (EWPI over LTI) is less in VP than in height fields.

Several colleagues (see acknowledgments) wondered whether EWPI is any different from spectral averaging (SA). Why not take a simple average of the phase and the amplitude of the wave 24 h earlier and later. Indeed, if waves moved slowly and regularly, that is, the speed is constant in time and equal to what is shown in Table 1, SA would accomplish exactly the same as EWPI. (In that case SA and EWPI would be somewhat like a wave-by-wave Hovmöller diagram approach.) However, some waves move fast and/or irregularly, and SA is inherently dangerous because the phase of a periodic wave is arbitrary by a multiple of 360°. The average of two phases therefore has (at least) two values that differ by 180°. SA (and LTI) do not address this ambiguity very well and often pick the wrong phase-averaged value. An error of 180° is the worst possible scenario, leading to the problems already shown in Table 3 for waves 7–12 when using LTI. SA is even worse than LTI because it keeps the amplitude of erroneously positioned waves very large, while LTI usually damps heavily when the wrong phase is chosen. The EWPI scheme on the other hand first moves waves to a common time, before averaging. At this common time the phase difference is smaller and the chances of picking the wrong phase average are greatly reduced. Also EWPI (as does LTI) averages the sine and cosine coefficients, not the phase and amplitude.

The demonstration in section 2 was for zonal harmonics only. We have repeated the whole exercise with spherical harmonics with qualitatively similar results. However, the gains of EWPI over LTI are less, the "reason" being that the speed of spherical harmonics, with zonal wavenumber $m = 6-12$, appears to be too slow for the midlatitudes and in the wrong direction in the Tropics. This proves that the choice of base functions does matter, and one wonders whether one can improve over zonal harmonics by some empirically defined more optimal functions (Brunet 1994).

All results presented were based on phase propagation in the zonal direction only. Some readers may wonder why meridional propagation was ignored. To our knowledge, there is no obvious definition of meridional phase speed. When using spherical harmonics, meridional (as well as zonal) group velocity becomes part of EWP.

In practice, more than one data point could be missing. We repeated the calculation, by interpolating from data 48 h before and after the (assumed) missing data point. This simulates a case where three successive daily values are missing. Of course the skill is less than before. The AC for LTI and EWPI now is 0.635 and 0.717, respectively, for 50°N, compared to 0.845 and 0.913 (see Table 2), when only one value was missing. So even with three values missing, accounting for wave motion helps and EWPI can still make a reasonable substitute. We repeated the calculation for 5, 7, etc. missings and found some slight benefit in using EWPI until as much as 11 successive daily data are missing; beyond this point skill becomes very small (AC < 0.2) no matter what scheme is used. We apparently have a situation in which even rather distant future and past data can be called upon for interpolation with some skill. Returning to a single missing data point, one might then ask whether data at 48, 72, etc. hours before and after the missing point, combined with data 24 h before and after could produce an improved interpolation. This question is related to four-dimensional data assimilation (Tanguay et al. 1995) using future as well as past data, and perhaps EWP, the adjoint of which can be written easily, has an application here.

We have shown that EWPI has skill as a time interpolation method. A higher-order achievement would be if EWPI also were to improve the accuracy of the time derivative. In many studies (Brunet et al. 1995; Cai and Van den Dool 1994) one needs to know the time derivative, and estimating it from 12- or 24-h data may not be very accurate, particularly for short mobile waves. It was noted by Cai and Van den Dool (1994), for instance, that the time derivative of relative vorticity (from data 24 h apart) was typically 50% smaller than the sum of all the other terms (vorticity advection, stretching, etc.) that are instantaneous time tendencies. Given the 500-mb daily height data set used above, we noted that, when using EWPI to obtain hourly data, considerable detail is added to the dataset, and the time derivative at midpoint is considerably underestimated when differencing the data 24 h apart. Because the time derivative is not measured as such, we can not rigorously verify that EWPI helps in estimating tendencies, but the evidence certainly points in that direction.

This article has focused mostly on the results—skill in terms of AC. For a map of a single forecast based by EWP$^+$ the reader is referred to Qin and Van den Dool (1996), where EWP forecasts are compared systematically to those made by explicitly integrating the anomaly vorticity equation numerically, including nonlinearities and a wavy basic state background flow. Because of linear wave dispersion alone, EWP is capable of doing rather nontrivial things, including apparent "development" and formation of new wave packets downstream of an initial anomaly. EWP is much like an inexpensive numerical model, albeit that no equations are used and the future time evolution of the present condition is derived from historical data.

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APPENDIX

How to Calculate Phase Speed

Wave speeds are calculated as follows: For an anomaly wave m_0, at latitude φ we have at successive times, 24 h apart, t and $t + 1$:

$$t\text{: } A \cos m_0 (x - \mathcal{E}) = a \cos m_0 x + b \sin m_0 x \quad \text{(A1)}$$

$$t + 1\text{: } A_1 \cos m_0 (x - \mathcal{E}_1) = a_1 \cos m_0 x + b_1 \sin m_0 x \quad \text{(A1a)}$$

After phase shifting over $\mathcal{E}$, (A1) and (A1a) become:

$$t\text{: } A \cos m_0 (x) = A \cos m_0 x + 0 \sin m_0 x \quad \text{(A2)}$$

$$t + 1\text{: } A_1 \cos m_0 [x - (\mathcal{E}_1 - \mathcal{E})] = c_1 \cos m_0 x + d_1 \sin m_0 x. \quad \text{(A2a)}$$

The rhs coefficients A, c_1, and d_1 are a function of time. Their time averages are denoted by $\overline{A}$, $\overline{c}_1$, and $\overline{d}_1$. (The time mean of a and b is zero by definition.)

Amplitudes of time-averaged phase shifted waves:

$$t\text{: } A_p = \overline{A} \quad \text{(A3)}$$

$$t + 1\text{: } A_{p1} = (\overline{c}_1^2 + \overline{d}_1^2)^{1/2}. \quad \text{(A3a)}$$

Phase angles of the time-averaged waves are:

$$t\text{: } \mathcal{E}_p = 0 \quad \text{(A4)}$$

$$t + 1: \mathcal{E}_{p1} = \arctan(\bar{d}_1/\bar{c}_1). \qquad (A5)$$

To apply EWP, it is enough to know $\mathcal{E}_{p1}$ as a function of wavenumber and latitude. Phase speed (in meters per second) can be obtained from the phase angle $\mathcal{E}_{p1}$ (in radians) as $c = \mathcal{E}_{p1} a_0 \cos(\varphi)\,(86\,400 m_0)^{-1}$ where a_0 is the radius of the earth.

Note that the amplitude information in (A3a) has not been used in EWP/EWPI as proposed here. The above is a specific application of the more general "phase-shifting" technique described in Cai and Van den Dool (1991).

With the above and a dataset, Table 1 can be reconstructed. In Table 1 we showed speeds for zonal waves $m = 1-10$ at latitudes 60°S–60°N. The reason for not showing high latitudes and harmonics ($11 < m <= 15$) is purely cosmetic, that is, the propagation speed is not unambiguously determined when a wave travels climatologically more than half its wavelength per sampling time. For instance, $m = 11$ at 50°S will travel 200° eastward in a day, or is it 160° westward? Nevertheless there is no problem to include higher harmonics and higher latitudes in the EWPI scheme. We also found that waves beyond 15 contribute little or nothing to the skill of interpolated *height* fields. This "truncation" could be different for other fields of course.

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