A unified theory for the Madden-Julian oscillation and the boreal summer intraseasonal oscillation

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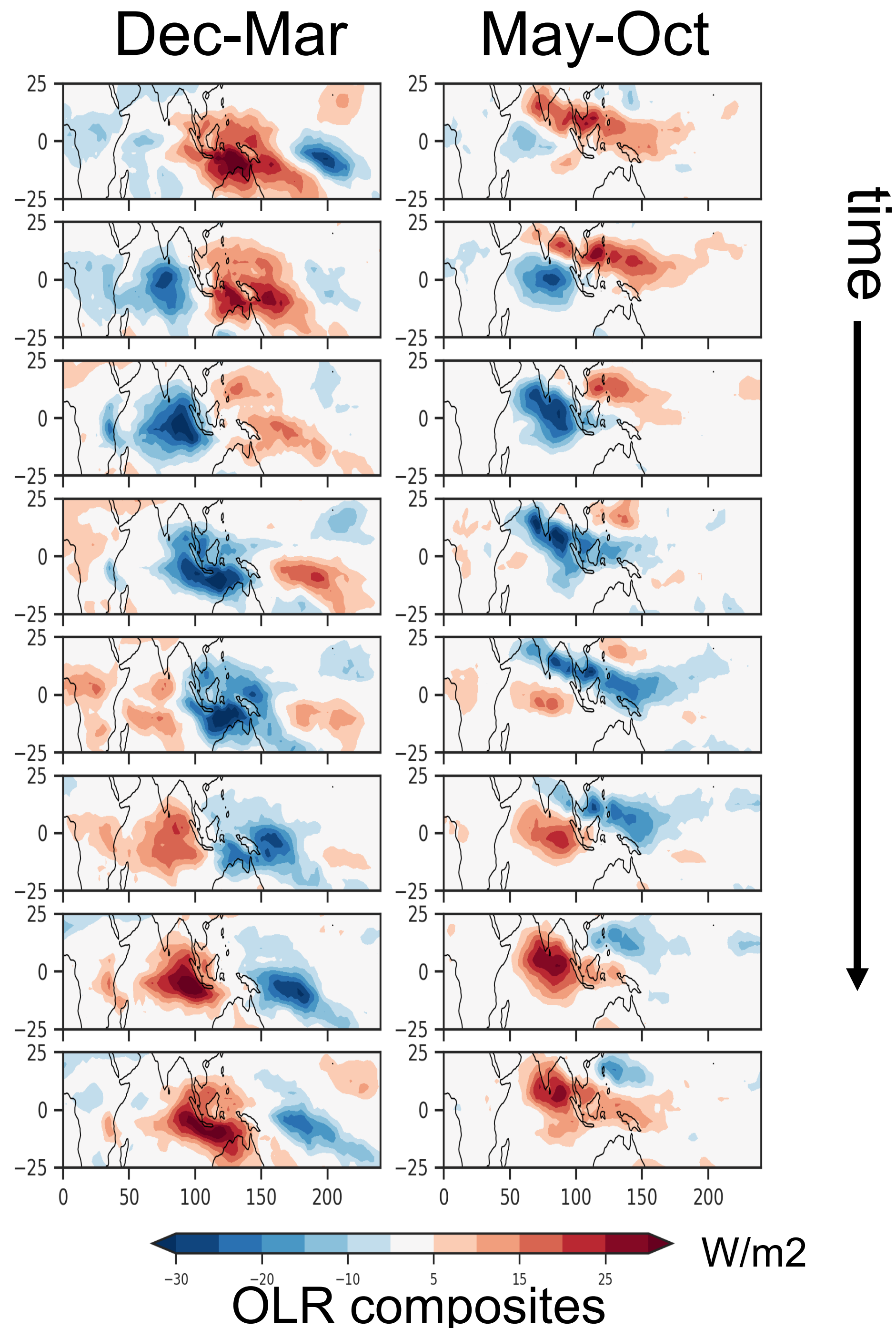
Madden-Julian Oscillation vs. Boreal Summer Intraseasonal oscillation

MJO

- northern winter
- eastward propagation
- Mostly the equator, biased toward SH

BSISO

- northern summer
- northwest-Southeast tilted rain bands
- significant northward propagation + eastward propagation



Short version

- We study an idealized linear model similar to those used in many previous studies
- The novel aspect is the inclusion of horizontal moisture advection in both the zonal and meridional directions, and consideration of varying meridional basic state moisture gradients
- The model solutions include “moisture modes” which qualitatively resemble the observed tropical intraseasonal oscillations
- **The solutions have a more MJO-like structure when the meridional moisture gradient is large (as in northern winter) and a more BSISO-like structure when the meridional moisture gradient is small, as in northern summer over the Indian Ocean**
- Thus this theory unifies the MJO and BSISO as different manifestations of the same thing, and specifies a mechanism controlling the change of structure with the season
- Similar to Ahmed (2021); what is different here are the parameters and consideration of different moisture gradients & seasonal cycle

Single vertical mode-> moist linear shallow water equations

$$\frac{\partial u}{\partial t} - yv = -\frac{\partial \Phi}{\partial x} - \alpha_u u$$

Rayleigh friction

$$\frac{\partial v}{\partial t} + yu = -\frac{\partial \Phi}{\partial y}$$

cloud-radiative feedback

Newtonian damping

$$\frac{\partial \Phi}{\partial t} + \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = -(\alpha + \varepsilon)q - \alpha_\Phi \Phi$$

precipitation/latent heating

$$\frac{\partial q}{\partial t} - (\Gamma - 1)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = \Lambda u - \alpha q - u\frac{\partial Q_o}{\partial x} - v\frac{\partial Q_o}{\partial y}$$

meridional moisture advection

Gross moist stability

WISHE

zonal moisture advection

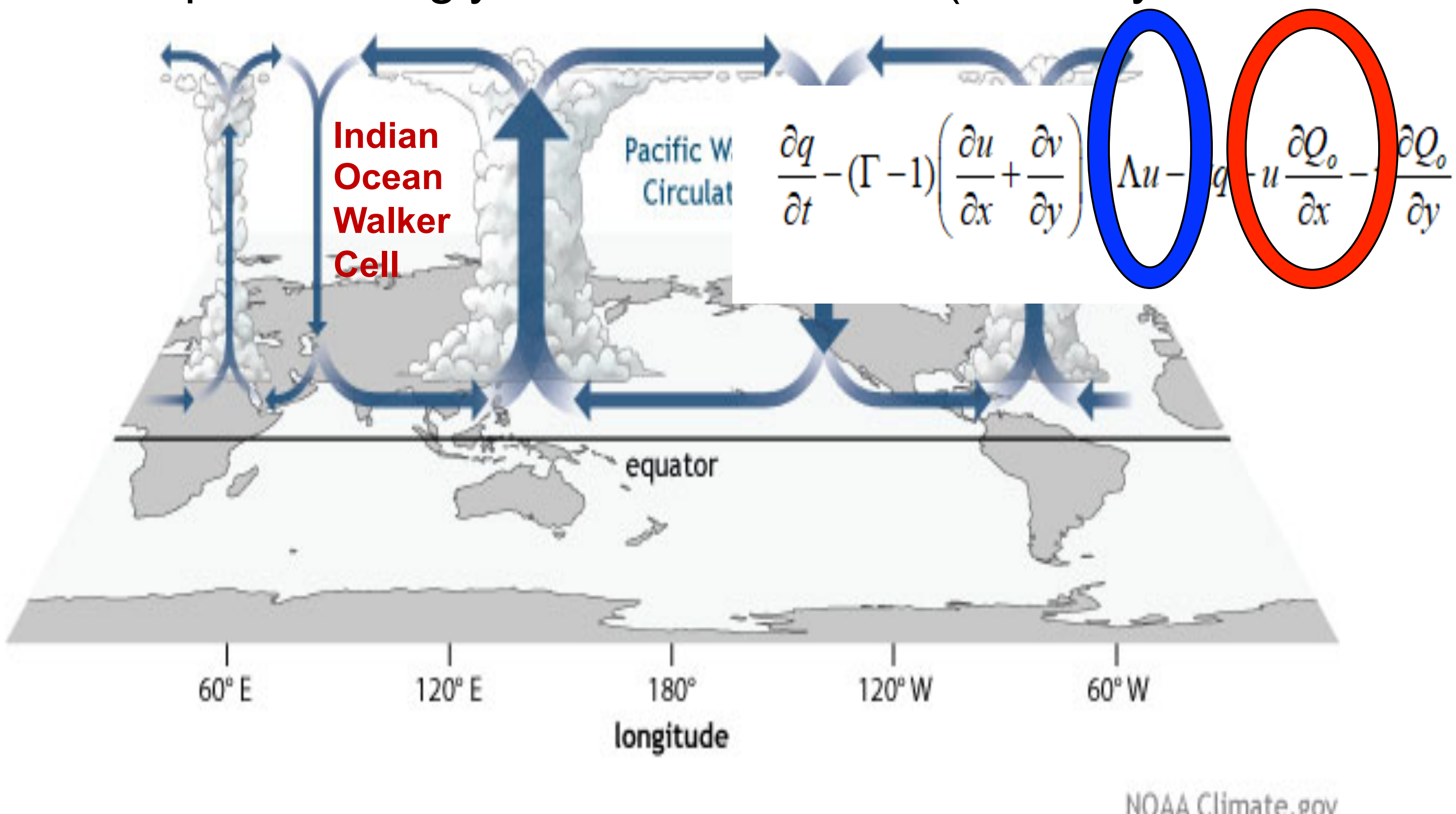
First baroclinic mode vertical structure

Simple quasi-equilibrium convection parameterization

8 parameters, 5 of them in the moisture equation

We assume a basic state with low-level westerlies and moisture increasing with longitude, as over the Indian Ocean. Thus we have:

- positive WISHE parameter Λ: low level westerly anomalies moisten by WISHE
- positive moisture gradient (Q_x) low level westerlies dry by zonal advection, $-u'Q_x$
- These terms both depend linearly on u, and so can be combined, the dynamics depend strongly on which one wins (Maloney and Sobel 2013)



Surface flux & horizontal advection anomalies vs. 850 hPa zonal wind Indian Ocean, 20-90d bandpass

$$\frac{\partial q}{\partial t} - (\Gamma-1)\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right) = \Lambda u - \alpha q - u\frac{\partial Q_o}{\partial x} - v\frac{\partial Q_o}{\partial y}$$

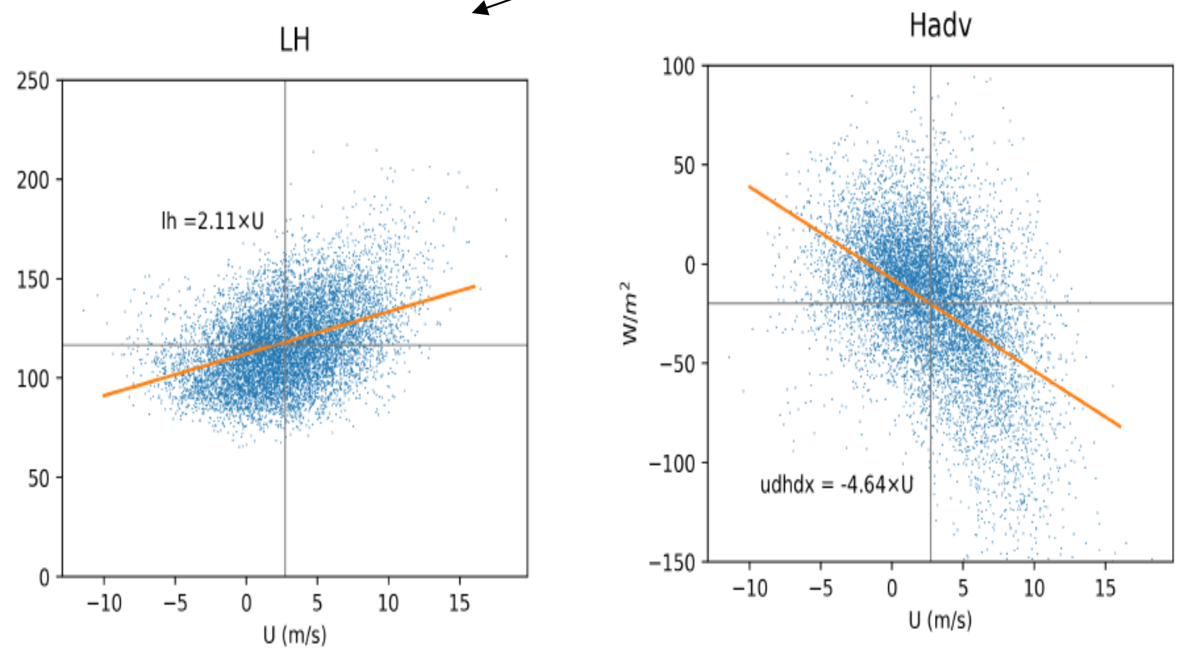


FIG. 1. (a) Column integrated zonal MSE advection vs U850 (zonal winds at 850 hPa); (b) Surface latent heat fluxes (LH) vs U850 using the ERA5 dataset. The three variables averaged between 70-80 °E and 5 °S - 5 °N. 5-day running average is applied to the time series to reduce noises. The gray bars indicate the mean values.

Zonal advection wins!

Assume wave solutions: v(x,y,t) ~ v(y) exp[i(kx-ωt)], obtain a single equation for *v* (as long as $v \neq 0$)

moist *v* equation

$$\boxed{a_0 \frac{\partial^2 v}{\partial y^2} + a_1 y \frac{\partial v}{\partial y} + (d_0 + d_2 y^2) v = 0}$$

dry *v* equation

$$\boxed{\frac{\partial^2 \hat{v}}{\partial y^2} + \left[\left(\frac{\nu^2}{g h_e} - k^2 - \frac{k}{\nu}\beta \right) - \frac{\beta^2 y^2}{g h_e} \right] \hat{v} = 0}$$

(Matsuno 1966)

The coefficients a_0, a_1, d_0 and d_2 are :

$$a_0 = \omega_u \left[(\Gamma - 1)(\alpha + \varepsilon) - i\omega_q \right]$$

$$a_1 = [i(\Lambda - Q_x) + \omega_u \bar{Q}](\alpha + \varepsilon)$$

$$d_0 = a_1 - \frac{k}{\omega_u} a_0 - k^2 \frac{\omega}{\omega_u} a_0 - i\omega\omega_q\omega_u\omega_\Phi + ik\omega(\Lambda - Q_x)(\alpha + \varepsilon)$$

$$d_2 = i\omega_q\omega_\Phi + k\bar{Q}(\alpha + \varepsilon).$$

where the damping coefficients are absorbed into ω as $\omega_q = \omega + i\alpha$, $\omega_u = \omega + i\alpha_u$,

cf. Fuchs and Raymond (2005); Ahmed (2021)

The dispersion relation is 8th order; it becomes 4th order under the longwave approximation

$$2a_1^2 + a_1 d_0 - d_0^2 - 9a_0 d_2 = 0$$

$$a_0 = \omega_u[(\Gamma-1)(1+r)\alpha - i\omega_\alpha]$$

$$a_1 = [i(\mathcal{E}-\overline{Q_x}) + \omega_u Q^0]\alpha(1+r)$$

$$d_0 = a_1 - \frac{k}{\omega_u}a_0 - k^2\frac{\omega}{\omega_u}a_0 - i\omega\omega_\alpha\omega_u\omega_\phi + ik\omega(\mathcal{E}-\overline{Q_x})\alpha(1+r)$$

$$d_2 = i\omega_\phi\omega_\alpha + kQ^0\alpha(1+r),$$

There are two kinds of solutions (as in the dry problem)

$$v \propto H_n\left(\frac{y}{\eta_n}\right)e^{-\xi_n y^2},$$ or v=0

$$v \propto H_n\left(\frac{y}{\eta_n}\right)e^{-\xi_n y^2}$$

The H_n are Hermite polynomials, sort of...

η_n and ξ_n are two new scaling factors. They are complex!

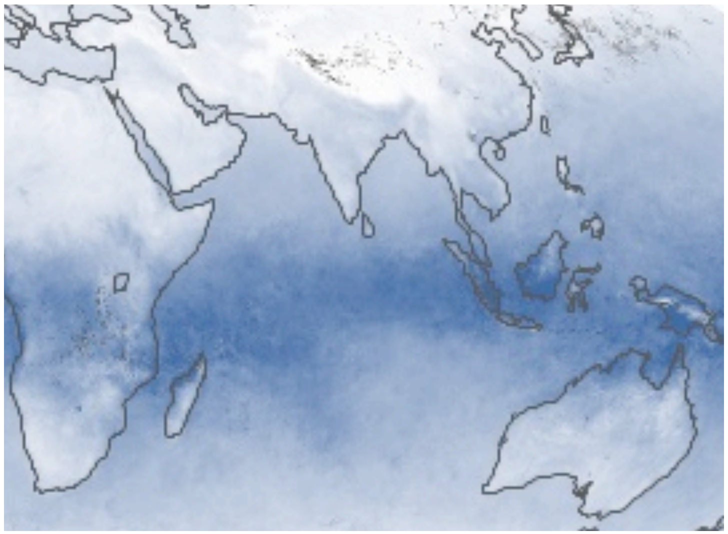
This implies oscillation in latitude, with local wavelength decreasing with latitude, even for *n*=1.

In canonical dry equatorial wave theory, this doesn't happen; the coeffs are real and fast oscillation with latitude only happens for large *n.*

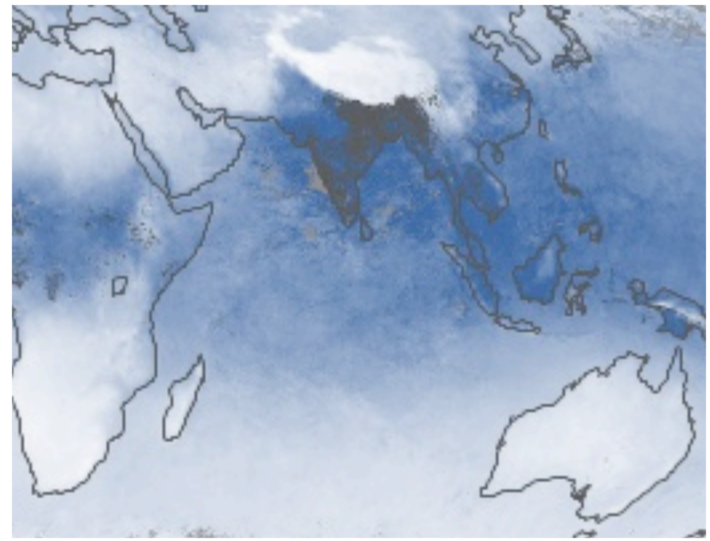
We discard solutions for which $\mathrm{Re}(\xi_1)<0$, implying wave grows in latitude.

Cf. Ahmed (2021)

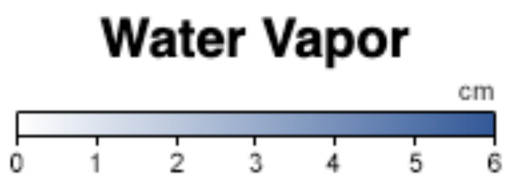
We consider the solutions as a function of the meridional moisture gradient in the Indian Ocean sector, Q_0.
Q_0 large -> NH winter; Q_0 small (or reversed) ->NH summer



January 2020

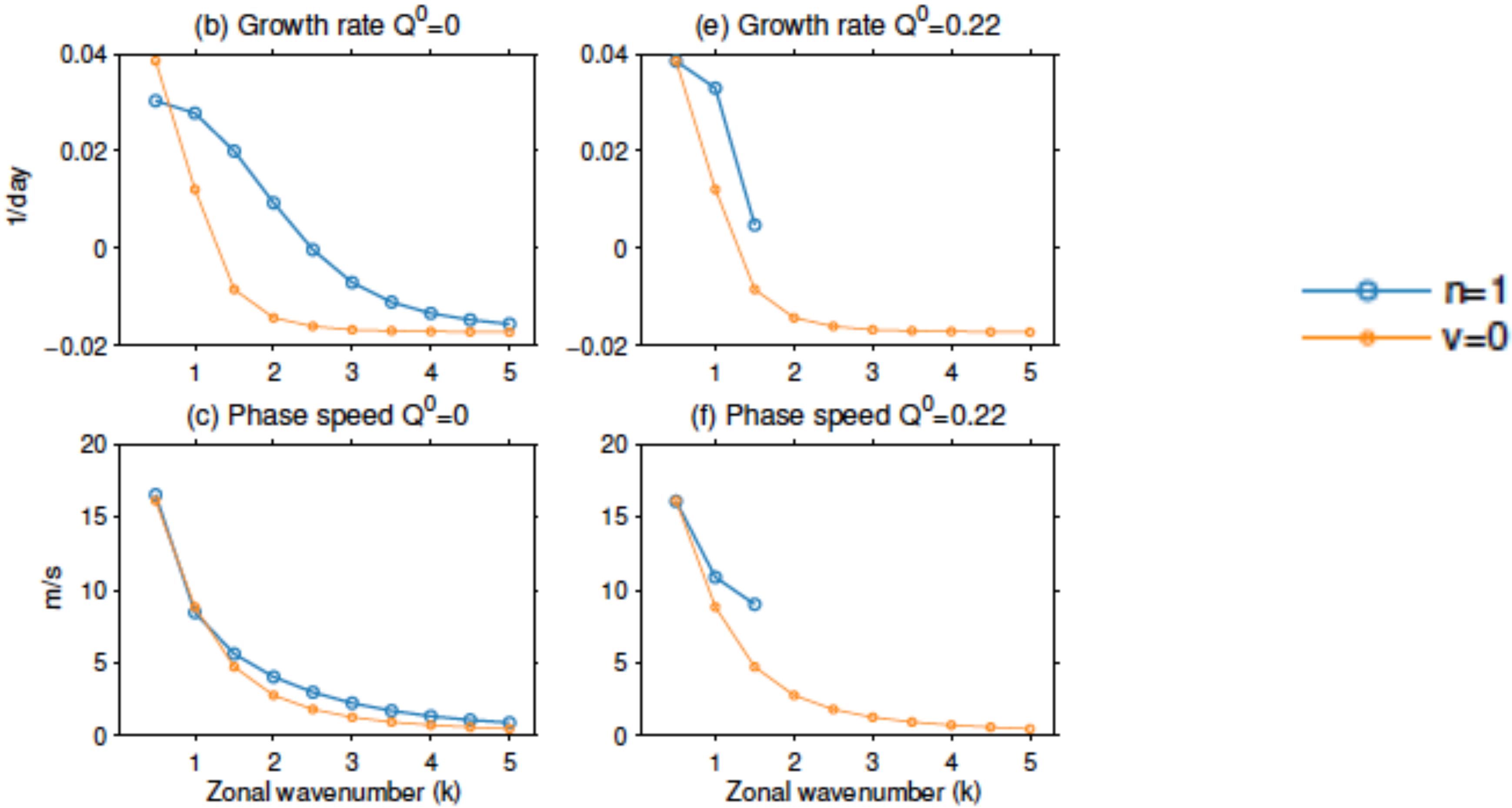


July 2020



https://earthobservatory.nasa.gov/global-maps/MYDAL2_M_SKY_WV

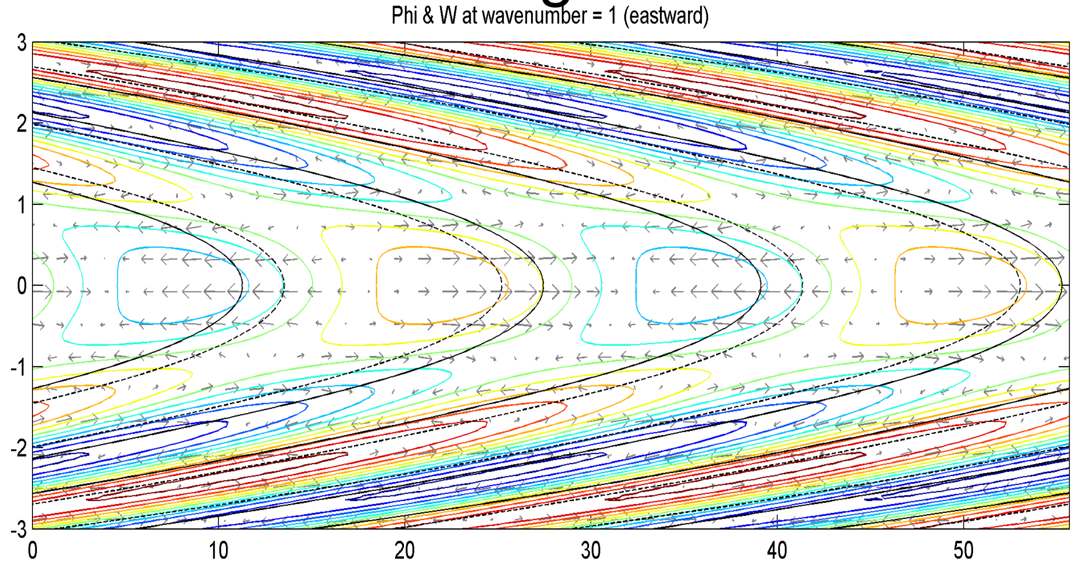
For both small and large meridional moisture gradient, max growth rates for large horizontal scale eastward-proagating modes



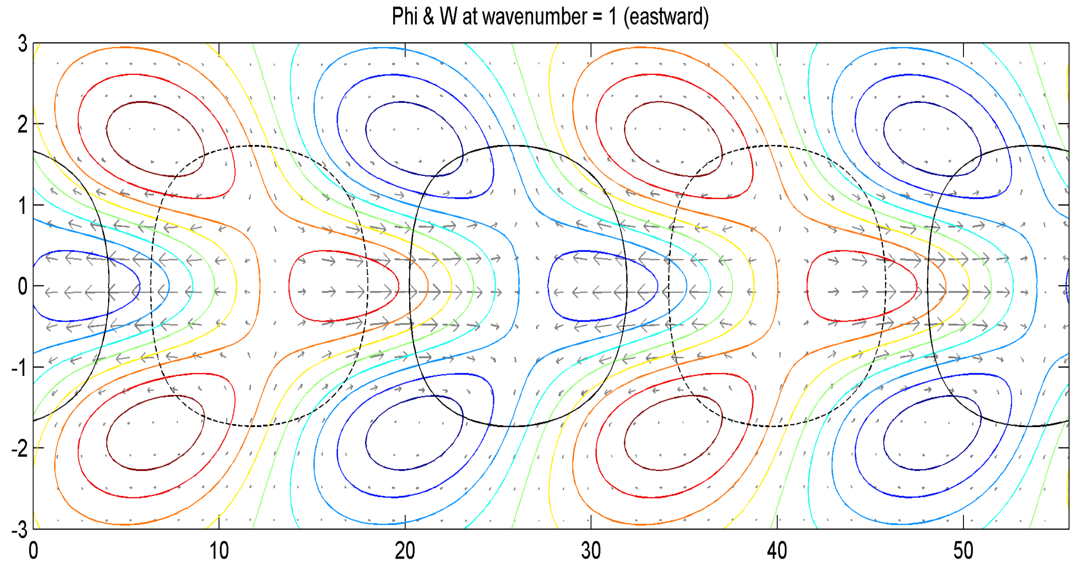
Dispersion

Change in structure with moisture gradient!

$Q_y = 0$
(like northern summer)



$Q_y = 0.22$
(like northern winter)



Color: geopotential
Black: vertical velocity
horizontal wind vectors

Change in structure with moisture gradient!

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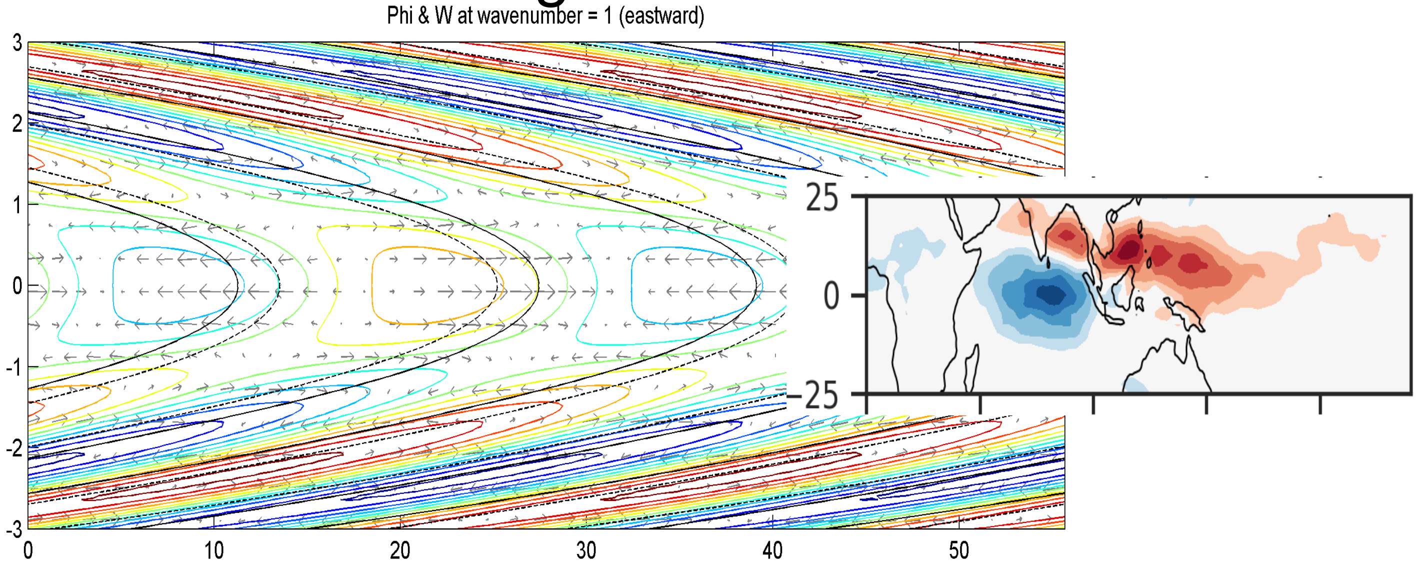

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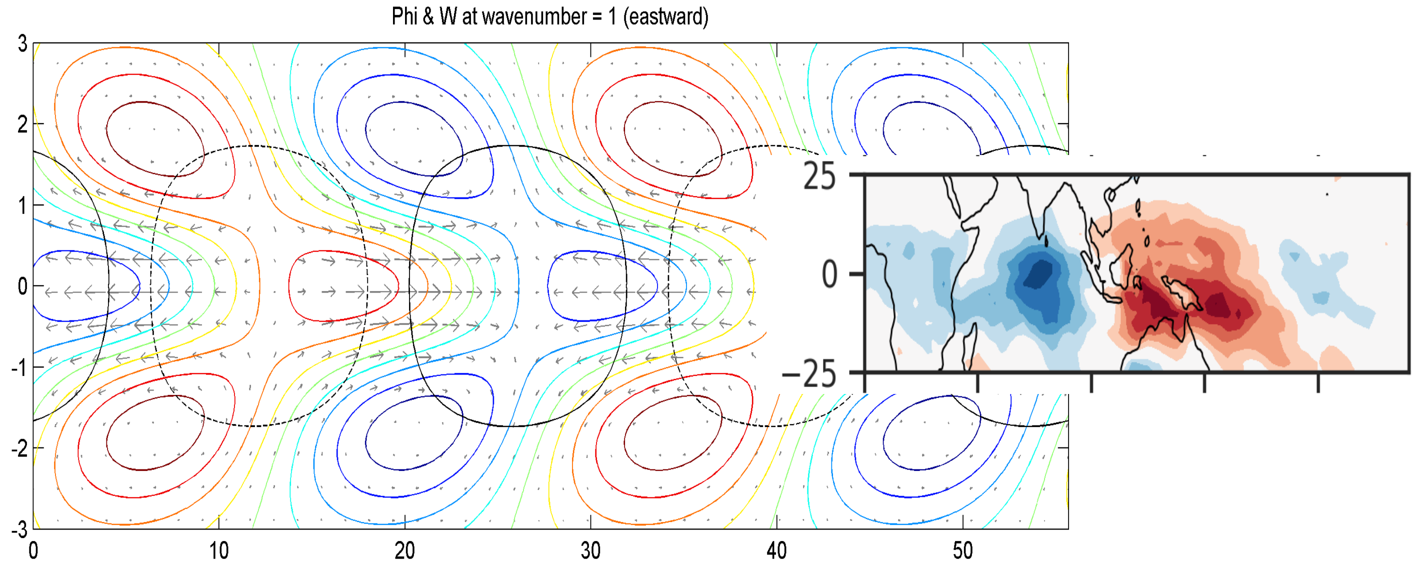

Color: geopotential
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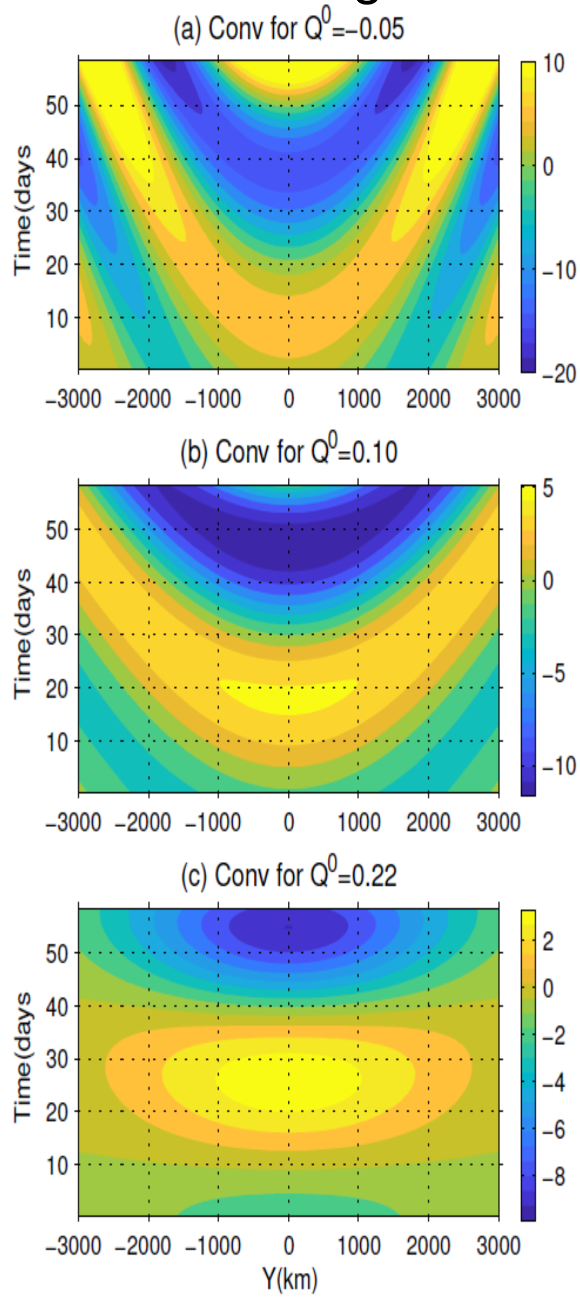
Latitude-time Hovmoeller plots show meridional propagation slowing and then stopping as the moisture gradient increases

BSISO

Increasing Q_y

MJO



Observed BSISO (Waliser et al. 200

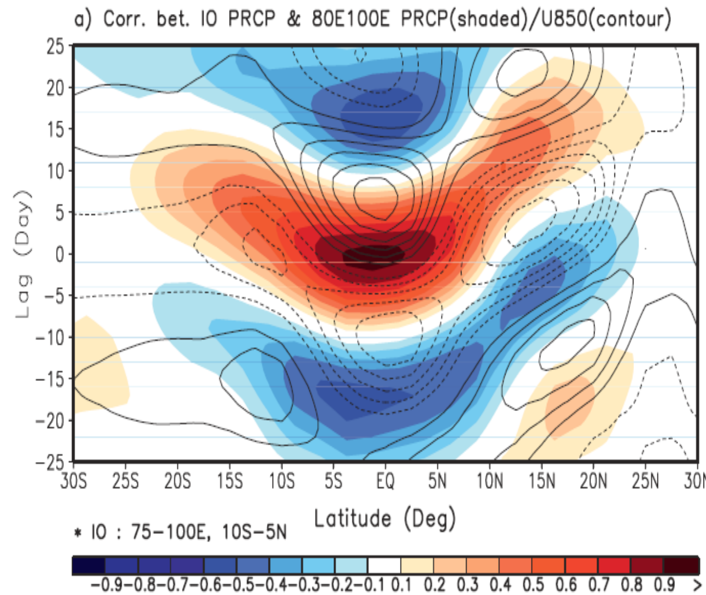


FIG. 6. May–October lag–latitude diagram of 80°–100°E-averaged intraseasonal precipitation anomalies (colors) and intraseasonal 850-hPa zonal wind anomalies (contours) correlated against intraseasonal precipitation at the Indian Ocean reference point at the equator. Contours and colors are plotted every 0.1. The zero line is not shown.

Key points

- This theory unifies the MJO and BSISO as different manifestations of the “moisture mode” over the Indian Ocean (where both modes grow)
- The different structures are a result of the different seasonal states of the climatological moisture field
- Horizontal advection of moisture − both zonal and meridional - is an essential process