# Space-Time Downscaling of Probabilistic Seasonal Forecasts with a "Weather Generator"

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**Abstract**

Stochastic daily weather time-series models ("weather generators") are parameterized consistent with both local climate and probabilistic seasonal forecasts. Both single-station weather generators, and spatial networks of coherently operating weather generators are considered. Only a subset of parameters for individual station models (proportion of wet days, precipitation mean parameters on wet days, and daily temperature means and standard deviations) are found to depend appreciably on the seasonal temperature and precipitation outcomes, so that extension to coherent multisite weather generators is straightforward. The result allows stochastic simulation of multiple daily weather series, conditional on seasonal forecasts. Example applications of spatially integrated extreme daily precipitation and snowpack water content are used to illustrate the method.

**1. Introduction**

Recent advances in understanding the climate system have allowed successful forecasts of seasonal temperature and precipitation at lead times up to a year in advance. At least two groups currently produce operational seasonal forecasts: the Climate Prediction Center (CPC) of the U.S. National Centers for Environmental Prediction (Barnston et al. 1999), and the International Research Institute (IRI) for Climate Prediction (Mason et al. 1999). Both the CPC and IRI seasonal forecasts are issued in a discrete, "tercile" format. That is, each forecast consists of a triplet of probabilities { $p_B$, $p_N$, $p_A$} pertaining to the three events "below-normal," "near-normal," and "above normal." Comparison of past seasonal forecasts with corresponding observed seasonal outcomes has demonstrated real and potentially useful information content (Wilks 2000a; Wilks and Godfrey 2000, 2002), but the temporally aggregated nature of the forecast quantities may be difficult for some decision makers to incorporate into their operations. In particular, many models of agricultural, hydrological, and other weather- and climate-sensitive managed systems operate on a daily time step.

**2. Weather generator**

A simple and successful way of representing the statistics of daily weather variations is the class of time-series models for surface weather data known as "weather generators" (Richardson 1981, Wilks and Wilby 1999). Weather generators are straightforward to fit to observed data, and the fitted parameters can be regarded as means of summarizing the surface climate of a location. These models are also easily linked to random number generation algorithms, to yield stochastic realizations of daily weather series that resemble real weather data with respect to a variety of relevant statistics. Furthermore, their mathematical structure is sufficiently simple that the implied seasonal statistics (to which seasonal forecasts pertain) can be computed from the parameters governing the daily stochastic weather processes (e.g., Katz 1985, Wilks 1992). Details of the single-station weather generator used here are provided in Wilks (2002).

Extension to spatially coherent weather generation can be accomplished by forcing collections of single-site generators with random numbers having the proper spatial correlation structure (Wilks 1998). It is useful and compact to parameterize these correlations, separately for the precipitation occurrence and precipitation amounts processes, according to station separation distances, using functions that produce positive definite (physically realizable) correlation matrices (e.g., Cressie, 1993).

**3. Weather generator parameters as functions of seasonal forecasts**

Briggs and Wilks (1996) presented a procedure to estimate climatological statistics for a broad range of subseasonal variables, conditional on seasonal forecast probabilities, by bootstrapping (Efron and Tibshirani 1993) the observed climatological record consistent with the forecast probabilities. For sufficiently simple statistics it is straightforward to make the computations analytically (Briggs and Wilks 1996, Crowley 2000).

Let $N_B$, $N_N$ and $N_A$ be the number of years in a climatological record for a given location and season in which either the temperature or precipitation was below-, near-, or above-normal, respectively. Imagine a bootstrapping procedure in which some large number *L* resamples are taken from this record with replacement according to the probabilities in a seasonal forecast $\{ p_B, p_N, p_A \}$ . The bootstrapped expected value (i.e., climatological average, conditional on the forecast) of a statistic *X* is then

$$\begin{aligned} E_{boot}[X] &= \lim_{L \to \infty} \frac{1}{L} \left[ \sum_{i=1}^{N_B} \frac{p_B L}{N_B} x_i^{(B)} + \sum_{i=1}^{N_N} \frac{p_N L}{N_N} x_i^{(N)} + \sum_{i=1}^{N_A} \frac{p_A L}{N_A} x_i^{(A)} \right] \\ &= \frac{L}{L} \left[ \frac{p_B}{N_B} \sum_{i=1}^{N_B} x_i^{(B)} + \frac{p_N}{N_N} \sum_{i=1}^{N_N} x_i^{(N)} + \frac{p_A}{N_A} \sum_{i=1}^{N_A} x_i^{(A)} \right] \\ &= p_B \bar{x}^{(B)} + p_N \bar{x}^{(N)} + p_A \bar{x}^{(A)} \quad , \end{aligned} \tag{1}$$

where, for example, $x_i^{(B)}$ is the statistic of interest from the *i*th below-normal year. See Wilks (2002) for fuller details.

The extension to coherent multiple-site weather generation requires estimation of spatial correlation functions for both precipitation occurrences and amounts. The existence of such dependence can be investigated by computing these correlations for subsets of years when both members of a station pair experienced the same type (below-, near-, or above-normal) of season. For the stations in New York state investigated here these correlations differ only slightly in aggregate, and will be assumed in the following to be independent of the seasonal precipitation forecasts.

## 4. Recovery of seasonal statistics

The foregoing development will be illustrated using daily temperature and precipitation observations from 1951-1996 over the same network of 25 locations in New York state as in Wilks (1998). They are distributed across an area of approximate dimension 500 km (east-west) by 100 km (north-south), centered near 42N, 76W. Results described in this section pertain to all twelve (January-February-March through December-January-February) seasons. One fundamental aspect of the performance of daily weather generators conditioned on seasonal forecast probabilities is that the proportion of synthetic outcomes in each of the three seasonal categories should agree with the forecast probabilities. Note that the original forecasts are assumed to be well calibrated, so the question addressed here is the extent to which calibration of the seasonal forecast probabilities is carried forward to the simulated daily series.

Figure 1 shows the results for seasonal precipitation, summarizing 10000 realizations each for the 300 combinations of the 25 stations and 12 seasons. For each location, season, and forecast probability, the proportion of seasonal precipitation outcomes in each of the three categories has been tabulated. These are displayed in Figure 1 in as boxplots for each forecast probability. The agreement between forecast probabilities and outcome relative frequencies is generally good, and indeed compares favorably with the reliability of the forecasts themselves (Wilks 2000a; Wilks and Godfrey 2000, 2002).

Figure 2 shows the corresponding results for temperature forecasts at the 11 stations in the network that report temperature data. For these simulations the precipitation forecast has been specified as $p_B = p_N = p_A = 1/3$ (daily temperature simulations are conditioned on series of simulated daily precipitation occurrences). Again the daily weather generators yield distributions of seasonal outcomes that are consistent with the proportions specified by each forecast.

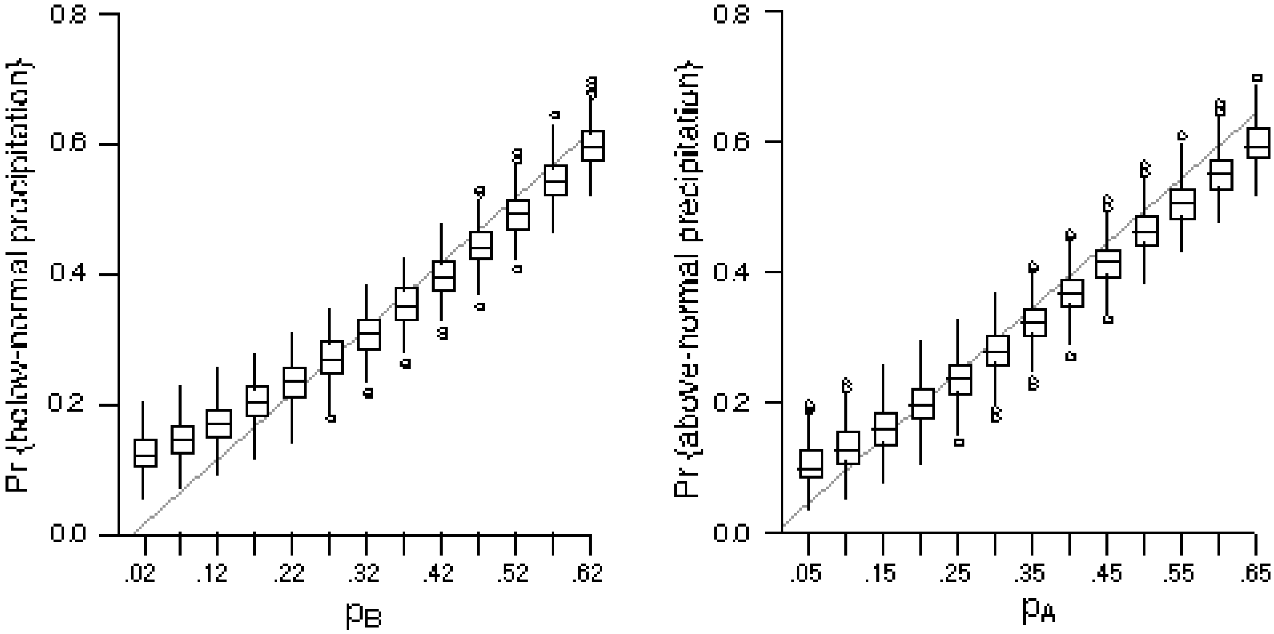


Figure 1. Distributions (over 25 locations and 12 seasons) of relative frequencies (in 10000 synthetic realizations each) of seasonal precipitation category outcomes, as functions of forecast probabilities constrained according to the CPC convention of $p_A = 2/3 - p_B$. The ideal 1:1 line is grey.

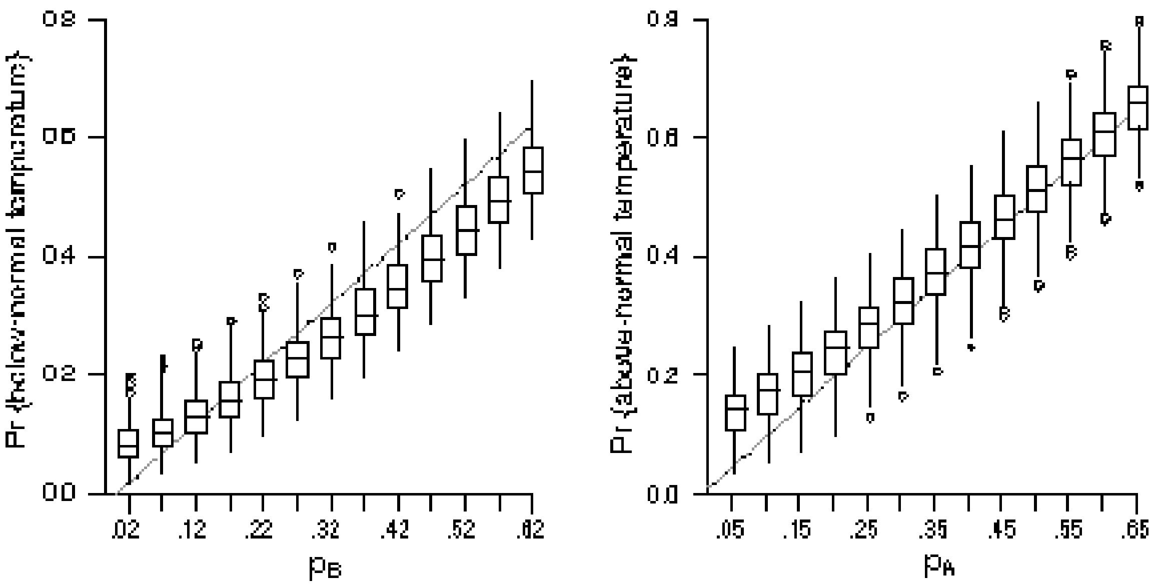


Figure 2. As Figure 1. but for seasonal temperature outcomes as functions of seasonal temperature forecast probabilities.

A useful diagnostic check for a daily weather generator is a comparison of its synthetic seasonal statistics with the seasonal statistics of the observations to which it has been fit (Gregory et al. 1993, Wilks and Wilby 1999). Figure 3 compares seasonal mean precipitation as simulated by daily weather generators (horizontal) with corresponding analytical calculations (Wilks 2000b) based on the climatological seasonal means and the seasonal forecast probabilities (vertical). In each case (25 stations x 12 seasons) there is very little scatter around the 1:1 line, indicating very good portrayal of the seasonal mean by the weather generators.

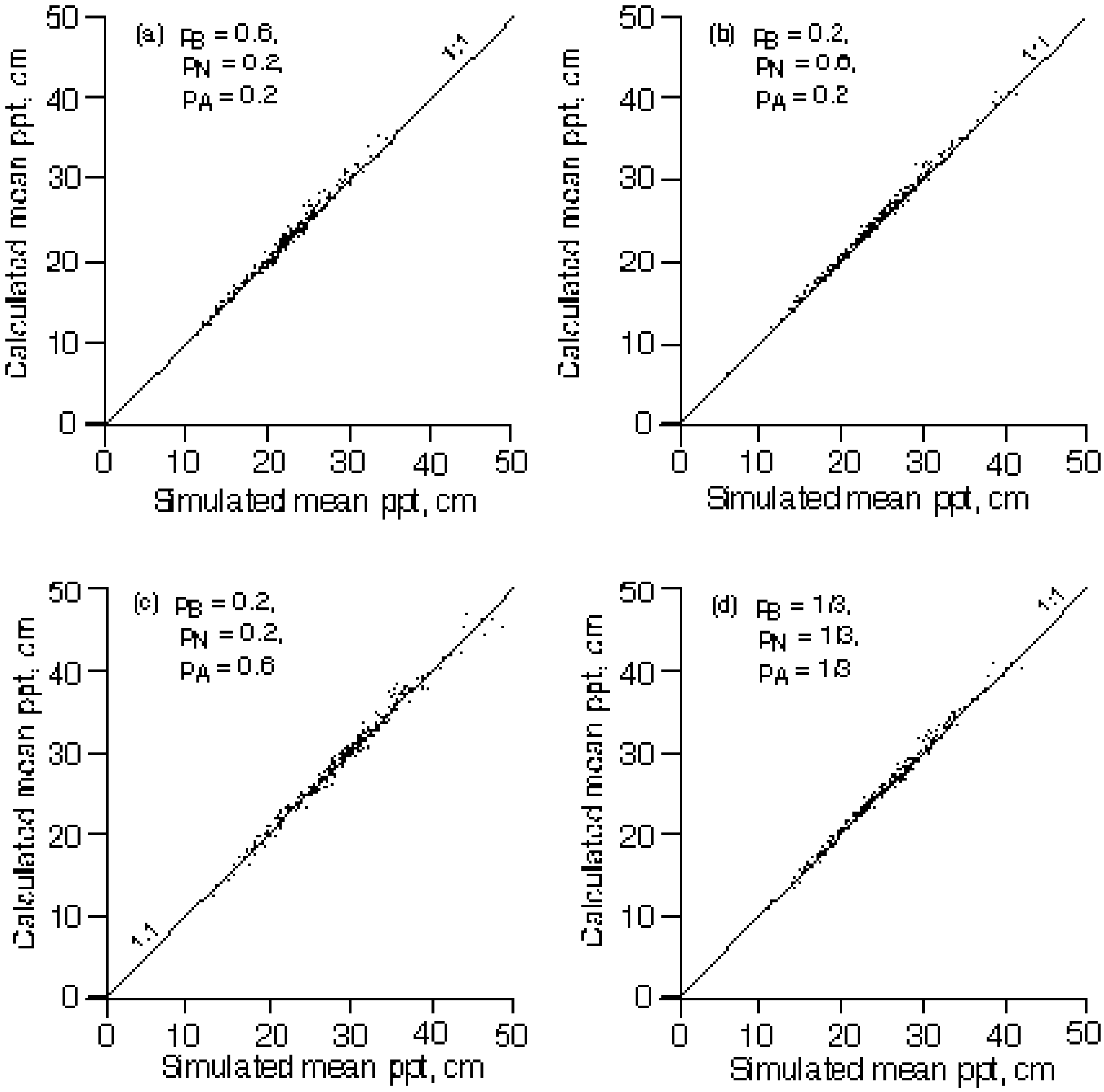


Figure 3. Comparisons of weather-generator simulated vs. calculated mean seasonal precipitation for: (a) a "dry" forecast, (b) a "near-normal" forecast, (c) a "wet" forecast, and (d) the climatological forecast; over 300 combinations of stations and seasons. The 1:1 line is drawn for comparison.

Figure 4 shows the corresponding results for standard deviations of seasonal precipitation, which describe the interannual variability of the seasonal precipitation totals. Panels (a), (c) and (d) exhibit the commonly observed "overdispersion" phenomenon (e.g., Katz and Parlange 1998, Wilks and Wilby 1999), i.e., the seasonal variations as simulated by the daily generators are smaller on average than their counterparts in the observations . In contrast, the interannual variations of seasonal precipitation implied by the daily generator for the "near-normal" forecast (b) are larger than the relatively small calculated values, which result is consistent with this modification of the weather generator parameters producing too many "dry" and "wet" seasons.

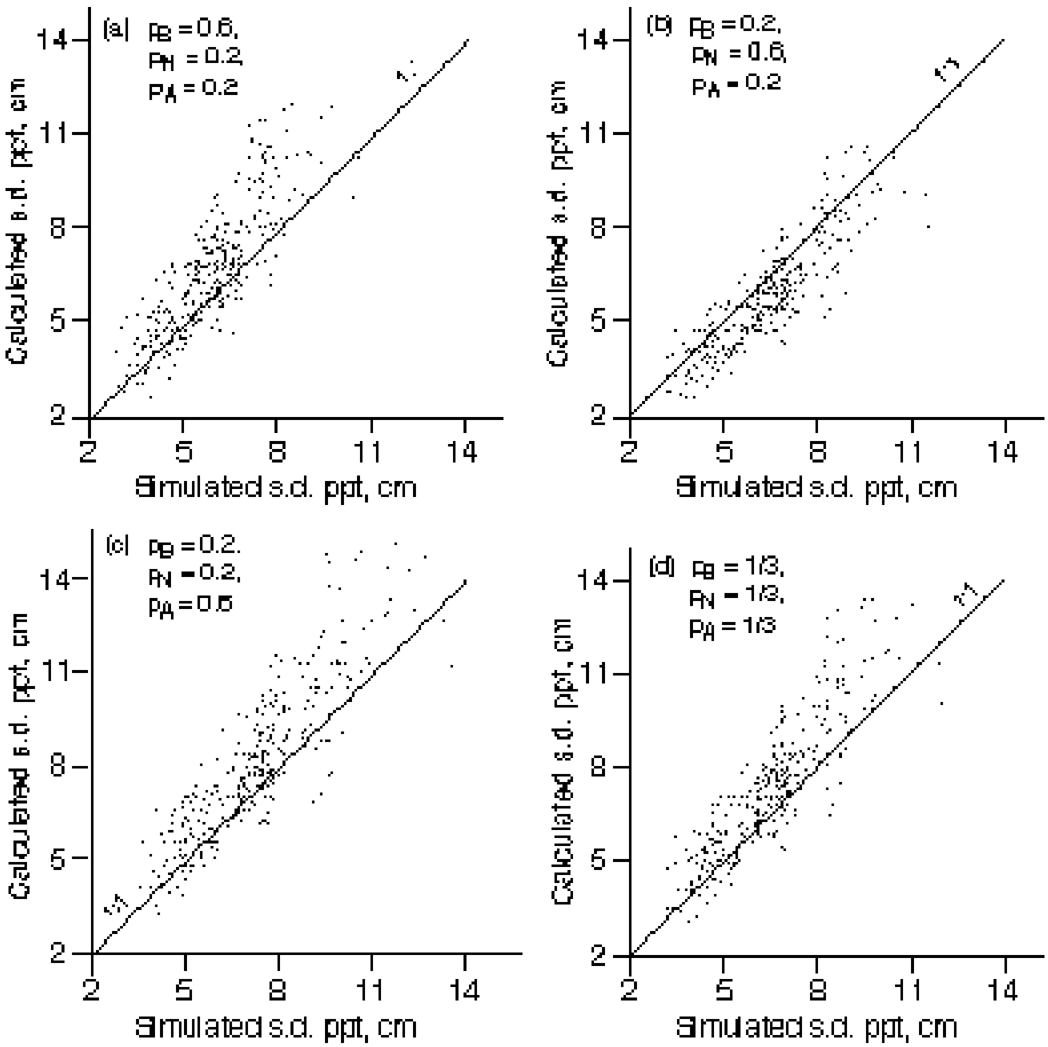


Figure 4. As Figure 5. for standard deviations of seasonal precipitation.

## 5. Two examples

### *a. Summer extreme precipitation*

While the New York stations considered in this paper are not part of a single watershed (they contribute to the Hudson, Delaware, Susquehanna, and St. Lawrence rivers), they will be treated in this subsection as if they comprised a single watershed, with each station contributing equally to an estimate of the "watershed" total precipitation.

Consider the average "return period" R, for an event of magnitude *X*. The return period is conventionally understood as the average time separating events of magnitude *X* or larger. In this subsection the event of interest *X* is the daily precipitation during the JJA season averaged over the entire "watershed," so that large values of *X* require large daily precipitation amounts simultaneously at a substantial fraction of the 25 stations.

The 46 years (1951-1996) of observed summer precipitation provide the data-based estimate for the basinwide return periods. Also considered are n = 10000-year realizations of spatially coherent daily precipitation corresponding to a "dry" forecast ($p_B$=0.50, $p_N$=0.35, $p_A$=0.15), a "wet" forecast ($p_B$=0.15, $p_N$=0.35, $p_A$=0.50) and the climatological ("CL") forecast ($p_B$=$p_N$=$p_A$=1/3). Figure 5 shows the results for average return periods between 2 and 1000 years. Circles indicate observed data values, the bold curve shows results for the CL forecast, and the light solid lines indicate the dry and wet forecasts. Results for the CL forecast agree reasonably well with the observed data, while simulated series based on the forecasts substantially affect the probabilities for the extreme outcomes. Also shown in Figure 5 are the results for the CL forecast, but with zero spatial correlation among the 25 simulated precipitation series. In this case very large precipitation amounts occur simultaneously at multiple stations with rather low probability, and the relationship to the observed extreme statistics is quite poor.

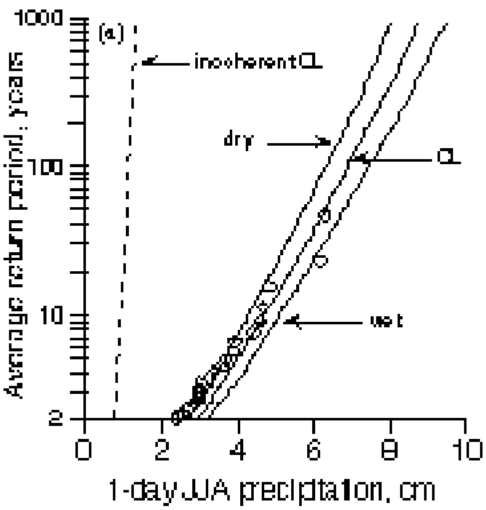


Figure 5. Average return periods for 1-day area-averaged precipitation calculated from the observed data 1951-1996 (circles); and 10000 synthetic years corresponding to "dry" ($p_B$=0.50, $p_N$=0.35, $p_A$=0.15) and "wet" ($p_B$=0.15, $p_N$=0.35, $p_A$=0.50) forecasts (light solid lines), and the climatological ($p_B$=$p_N$=$p_A$=1/3) forecast (heavy solid line). Dashed ("incoherent") lines show corresponding return periods for the climatological forecast when the spatial correlations among the 25 time series are set to zero.

*b. Winter snowpack water equivalent*

Another quantity of hydrological significance is the snowpack water equivalent (SWE), or liquid-equivalent water content of the snow and ice on the ground. SWE is not routinely measured at the Cooperative Observer stations, but will be modeled as a function of daily precipitation and temperature using the degree-day model of Carr (1988), which specifies 0.366 cm of snowmelt for each degree Celsius of average daily ( $[T_{max}+T_{min}]/2$ ) temperature above 0û C. This relationship was developed in Ontario, Canada, and has been found to be useful for quality-control of measured SWE data in the northeastern United States (Schmidlin et al. 1995). The following simple model of SWE dynamics will be used:

$$
SWE(t) = \begin{cases} SWE(t-1) + ppt(t) & , \quad \bar{T}(t) \le 0°C \qquad (2a) \\[2ex] \max\left[0,\ SWE(t-1) - 0.366\,\bar{T}(t) - \dfrac{T_{min}(t)}{T_{max}(t)}\,ppt(t)\right] & , \quad 0° < \bar{T}(t) \le \dfrac{T_{max}(t)}{2} \qquad (2b) \\[2ex] \max\left[0,\ SWE(t-1) - 0.366\,\bar{T}(t)\right] & , \quad \dfrac{T_{max}(t)}{2} < \bar{T}(t) \qquad (2c) \end{cases}
$$

Figure 6 shows extreme-value statistics for SWE during the DJF season, areally averaged over the 11 stations that have both temperature and precipitation data. Circles show modeled SWE using the 45 winters (1951/52—1995/96) of observed temperature and precipitation data. The solid lines show results for 10000-year spatially coherent weather generator simulations using combinations of "dry" or "cool" ($p_B$=0.50, $p_N$=0.35, $p_A$=0.15), and "wet" or "warm" ($p_B$=0.15, $p_N$=0.35, $p_A$=0.50) forecasts; or the climatological ($p_B$=$p_N$=$p_A$=1/3) forecast. Simulations based on the CL forecast (heavy line) reproduce reasonably the results calculated from observed data.

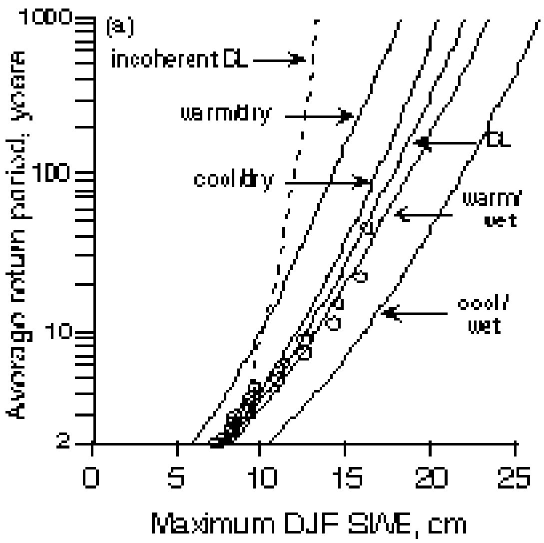


Figure 6. As Figure 5, but for maximum DJF simulated snowpack water equivalent (SWE). "Cool" forecasts indicate $p_B$=0.50, $p_N$=0.35, $p_A$=0.15, and "warm" forecasts indicate $p_B$=0.15, $p_N$=0.35, $p_A$=0.50.

The combinations cool/dry and warm/wet nearly compensate in terms of the SWE extremes. In contrast, the warm/dry and cool/wet combinations produce order-of-magnitude changes in extreme SWE return periods. The dashed lines in Figure 6 show corresponding results for spatially independent CL forecasts, which yield SWE extremes that are very much too light.

## 6. Conclusion

This paper has presented a method to adjust the parameters of daily time series models for weather data – weather generators – in a way that is consistent both with the observed climate of a location and seasonal forecasts in the format that is currently available operationally. It was found that only a subset of the weather generator parameters are sensitive to changes in the seasonal climate implied by the forecasts. For the precipitation submodel, only the unconditional probability of precipitation on a given day and parameters controlling the mean precipitation on wet days are sensitive to variations in the seasonal precipitation forecast. In the data considered here, only the daily temperature means and standard deviations were found to be sensitive to variations in the seasonal temperature forecasts. Correlations controlling time dependence of simulated weather series at individual locations varied to a much smaller degree, and in practice the unconditional climatological correlations could be used. Similarly, the correlations required to construct spatially coherent networks of weather generators were not sensitive to different forecast probabilities, and unconditional values for these parameters could be used as well. Before using the procedure described here for other regions, the validity of these results should be checked. While the examples have assumed the same seasonal forecasts throughout the domain, insensitivity of the spatial correlation functions to the seasonal forecasts implies that real applications could involve different forecast probabilities at different sites.

While subseasonal statistics consistent with a particular seasonal forecast can be estimated easily through bootstrapping (Briggs and Wilks 1996), for simple statistics equivalent analytic calculations are faster, more accurate, and no more difficult to implement. These calculations can be carried out on an as-needed basis, or (as has been done here) once and for all, through summary functions. First-moment statistics (mean fraction of wet days $\pi$, and mean precipitation amounts and temperatures) are planar functions of the forecast probabilities, while second-moment statistics (standard deviations and correlations) are quadratic surfaces.

Finally, note that use of the procedures described here is predicated on the seasonal forecasts being well calibrated ("reliable"). That is, relative frequencies of the event outcomes, conditional on the forecast probabilities, need to be essentially equal to the forecast probabilities in order for these procedures to be valid. Limited experience to date has found notable deficiencies in the calibration of seasonal forecasts (Wilks 2000a; Wilks and Godfrey 2000, 2002). Both the science and practice of seasonal forecasting continue to improve, but the forecast products should be used carefully.

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